Content

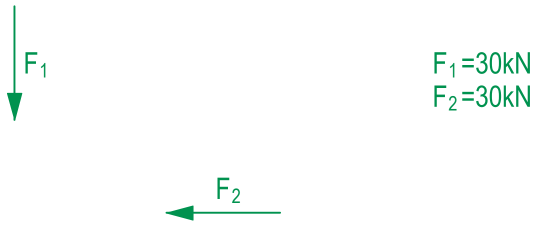
1.1

Compendium Structural Design I&II

Resultant of two non-parallel Forces

Two non-parallel forces F_1 and F_2 are given. The magnitude of the resultant and its position in the form diagram is to be found.

The resultant is the vector addition of all forces acting on the system. In graphic statics, the properties of the vectors (forces) are shown graphically in two different drawings. In the form diagram the position and the direction of the acting forces as well as the geometry of the structure are shown in a reduced scale. In the force diagram, however, the direction and magnitude of the forces are shown.



The two acting forces are drawn one after the other (clockwise) in the force diagram. The direction of the forces is maintained, their length in the force diagram corresponds to the magnitude of the force. This length is determined by the scale of the force diagram, where one centimetre [cm] corresponds to a certain number of kilo newtons [kN]. With the given scale, the vector of force F_1 in the force diagram is 3 cm long. F_2 begins at the end of F_1, also with a length of 3 cm. Together they form the so-called load line, i.e. the sum of all acting forces. The connection between the beginning of the first force and the end of the last force of the load line corresponds to the resultant R. The force diagram indicates the direction and the magnitude of the resultant.

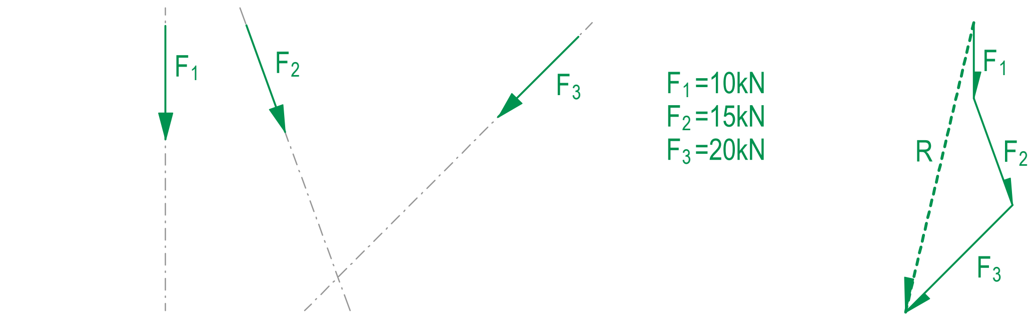
To find the position of the resultant in the form diagram, the lines of action of the attacking forces are drawn. The line of action of the resultant runs through their point of intersection. The direction of the resultant can now be moved parallel from the force diagram to the intersection point found in the form diagram. The position of R on the line of action, as well as the length of the vector, is freely selectable, since only the position and direction of the force, but not its magnitude, is shown graphically in the form diagram.

form diagrams 1:100

force diagrams 1cm ≙ 10kN

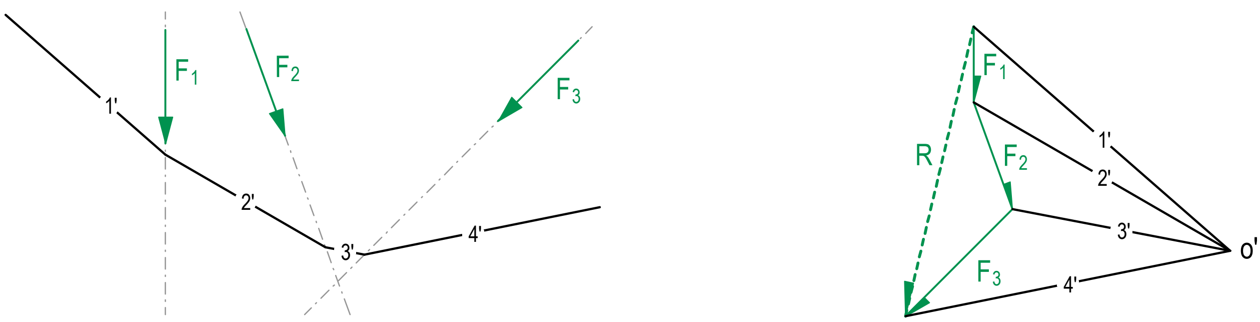
1.2 Resultant of several non-parallel Forces

Three forces of different directions are given. The magnitude and the position of the resultant of these three forces is searched for. The acting forces are shifted parallel one after the other (clockwise) in the force diagram and drawn together with the correct length using the given scale. The connection between the starting point and end point of the load line gives the magnitude and the direction of the resultant R.

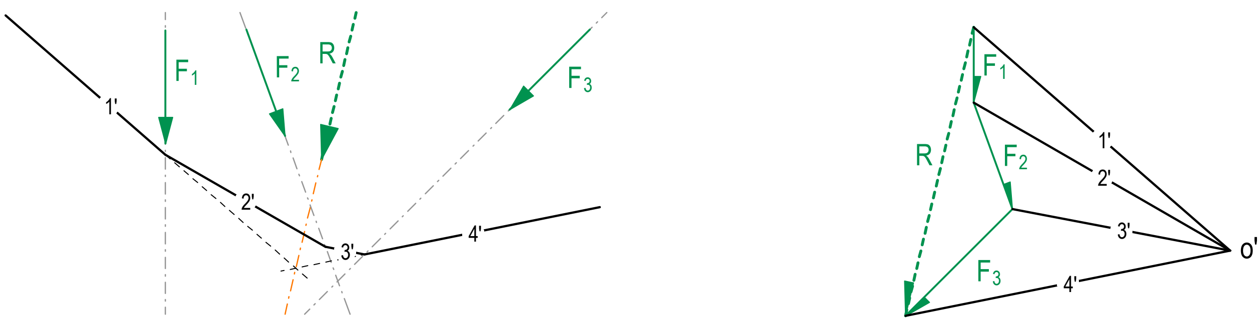


The position of the resultant in the form diagram is determined with the aid of a trial funicular. For this purpose a point o‘ is freely selected in the force diagram. Starting from this so-called pole, lines are drawn to the starting and end points of the forces F_1 to F3. These lines 1‘ - 4‘, the so-called rays, are now transferred in parallel into the form diagram. Since the form and force diagram are dual drawings each polygon in the force diagram corresponds to a point in the form diagram – and vice versa. The polygon F_1-1‘-2‘ in the force plan must therefore result in a point of intersection between the line of action of the first force and the first two rays.

The position of 1‘ can be freely selected in the form diagram. At the intersection point of 1‘ and F_1, 2‘ is then applied. Further the elements of the polygon F_2-2‘-3‘ is moved parallel. At the point of intersection of 2‘ with F_2, 3‘ is placed. This is continued until all rays have been transferred to the form diagram.



In order to find the position of the resultant, the first and the last segment of the trial funicular (1‘ & 4‘) are extended until they intersect. The angle of the resultant can now be moved parallel from the force diagram to this intersection point found in the form diagram. The position of the resultant always remains the same, even if a different pole o‘ is selected.



form diagrams 1:100

force diagrams 1cm ≙ 10kN

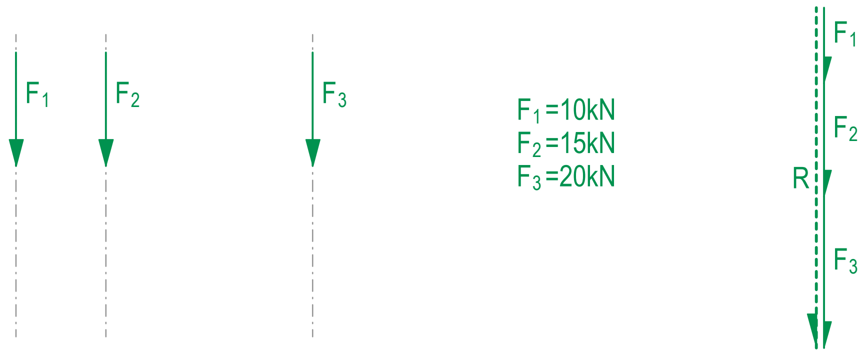
1.3

Compendium Structural Design I&II

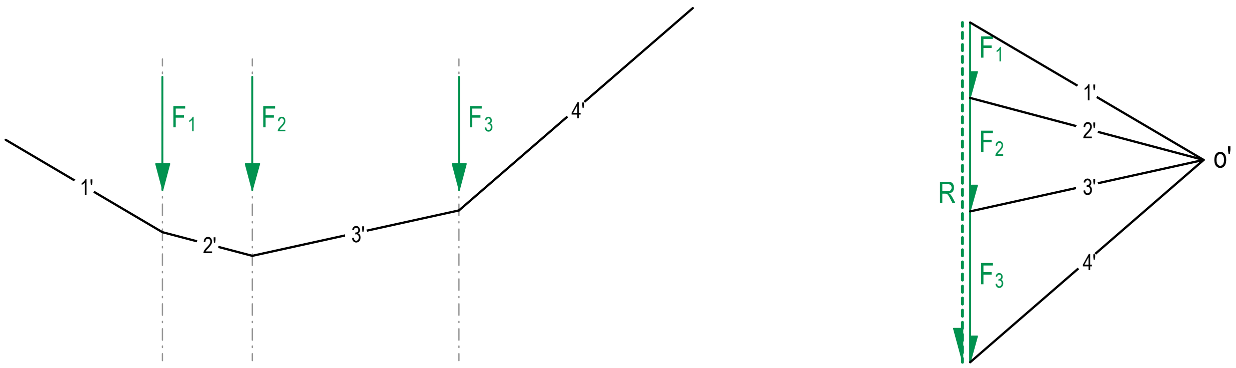
Resultant of several parallel Forces

Three parallel forces are given. The magnitude of the resultant and its position in the form diagram is searched for.

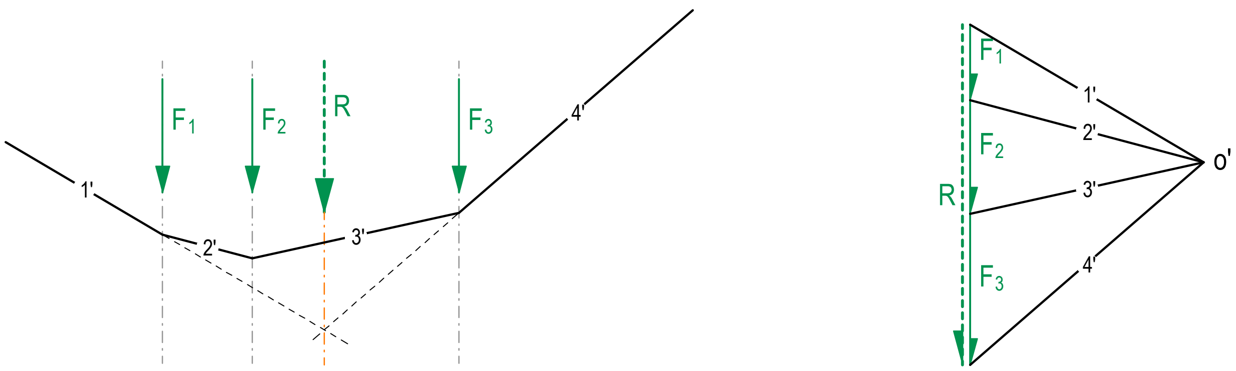
The acting forces are drawn one after the other (clockwise) in the force diagram using the given scale of the force diagram. The connection between the starting point and end point of the load line shows the magnitude and the direction of the resultant R.



The position of the resultant in the form diagram is determined with the aid of a trial funicular. For this purpose a point o‘ is freely selected in the force diagram. Starting from this so-called pole, lines are drawn to the starting and end points of the forces F_1 to F3. These lines 1‘ - 4‘, the so-called rays, are now transferred in parallel into the form diagram. Since the form and force diagram are dual drawings each polygon in the force diagram corresponds to a point in the form diagram – and vice versa. The polygon F_1-1‘-2‘ in the force plan must therefore result in a point of intersection between the line of action of the first force and the first two rays.
The position of 1‘ can be freely selected in the form diagram. At the intersection point of 1‘ and F_1, 2‘ is then applied. Further the elements of the polygon F_2-2‘-3‘ is moved parallel. At the point of intersection of 2‘ with F_2, 3‘ is placed. This is continued until all rays have been transferred to the form diagram.



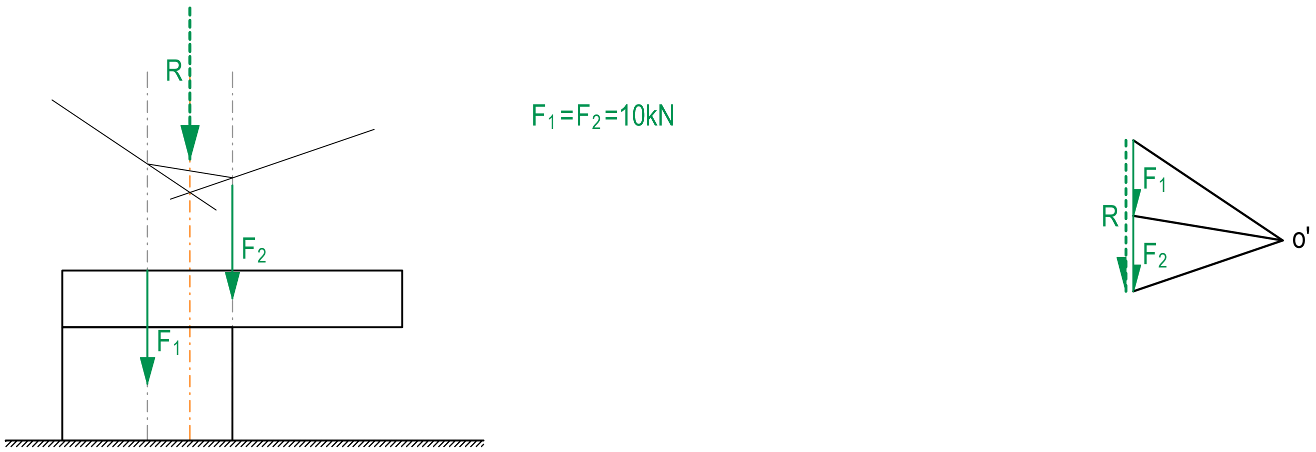
In order to find the position of the resultant, the first and the last segment of the trial funicular (1‘ & 4‘) are extended until they intersect. The angle of the resultant can now be moved parallel from the force diagram to this intersection point found in the form diagram. The position of the resultant always remains the same, even if a different pole o‘ is selected.



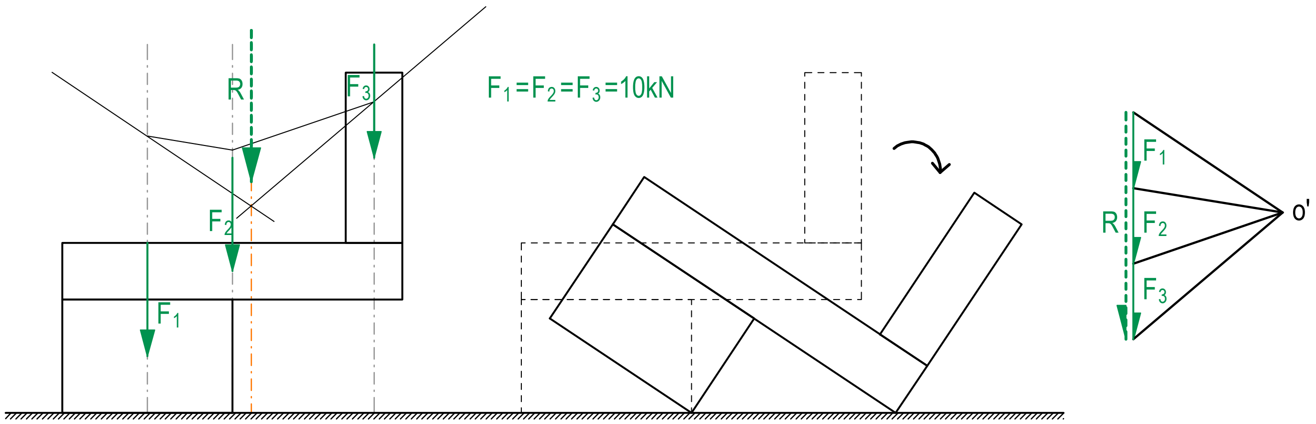
form diagrams 1:100 *force diagrams 1cm ≙ 10kN*

Three examples of differently stacked, glued together blocks are given. We are looking for the position of the resultant in the form diagram, in order to be able to judge whether the arrangement of the blocks is stable.

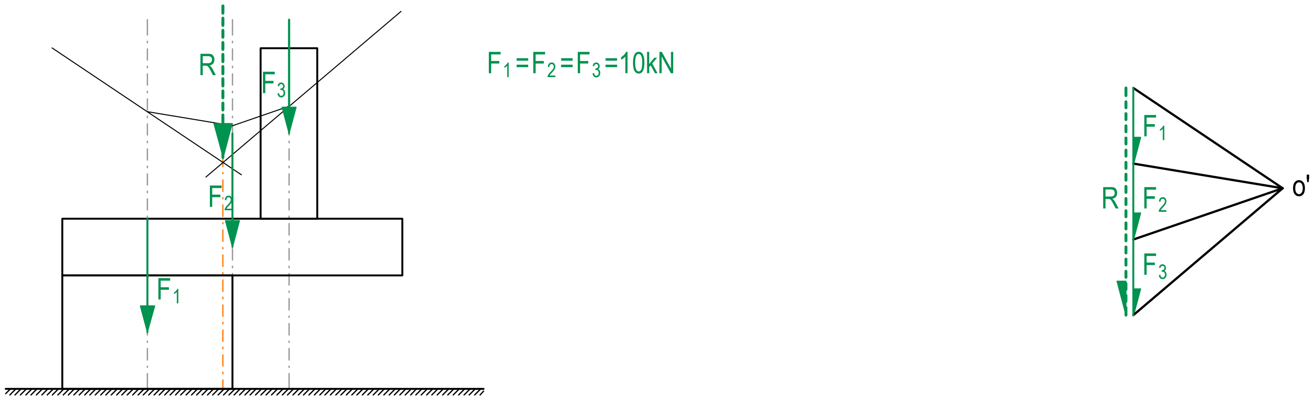
By means of an auxiliary construction, the position of the resultant is determined. Since the line of action of the resultant lies within the lowest building block, the construction is stable.



In the second example, the position of the resultant is again determined by means of an auxiliary construction. In this case the line of action of the resultant lies outside the lower block, which is why the construction would tip over to the right.



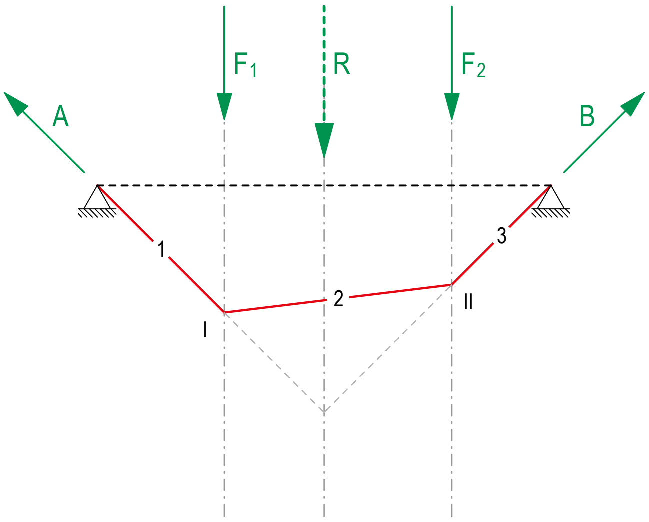
The previous example can be stabilized by moving the top block to the left until the line of action of the resultant is inside the bottom block again.



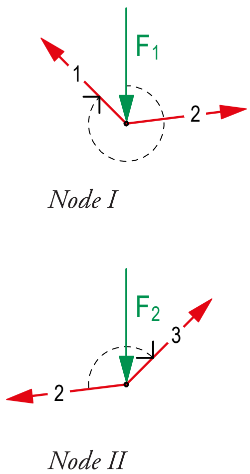
form diagrams 1:100 *force diagrams 1cm ≙ 10kN*

Form Diagram – Subsystem – Force Diagram

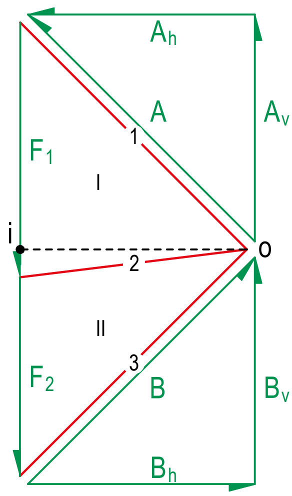
In graphic statics, the forces of a structure are displayed as vectors in two diagrams, the form diagram and the force diagram. The form diagram shows the geometry of the structure with all bearing elements and the position of the loads. The forces at and in the supporting elements are shown in the force diagram. Each line in the form diagram corresponds to a parallel line in the force diagram. The subsystems serve as sketches and show information regarding the individual nodes.



Form Diagram 1:100



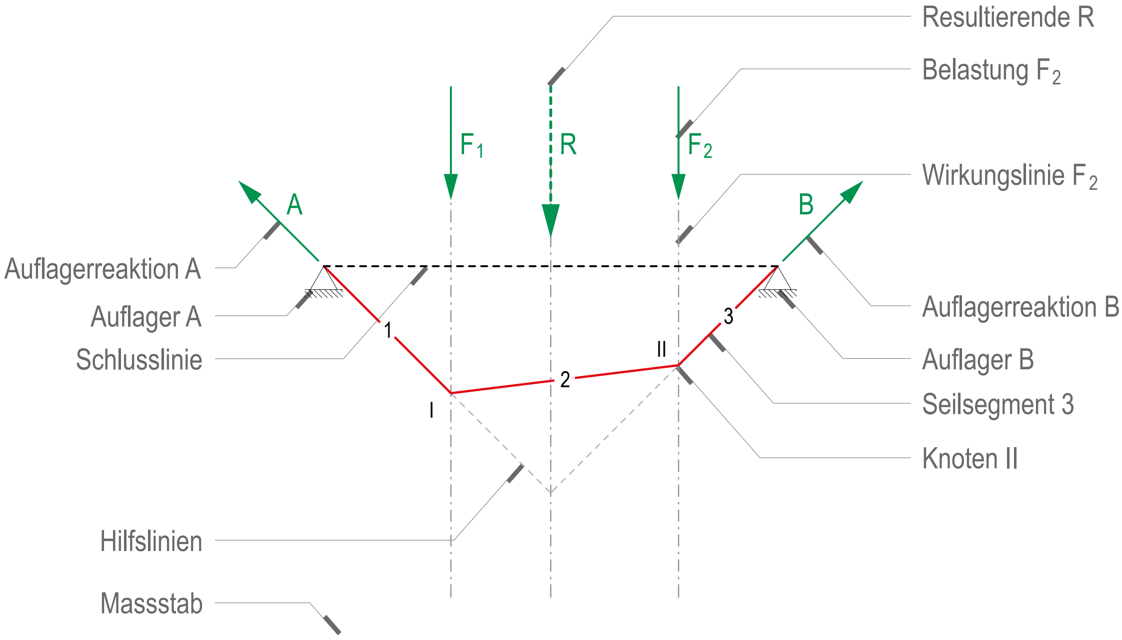
Subsystem (without scale)



Force Diagram 1cm ≙ 10kN

Form diagram

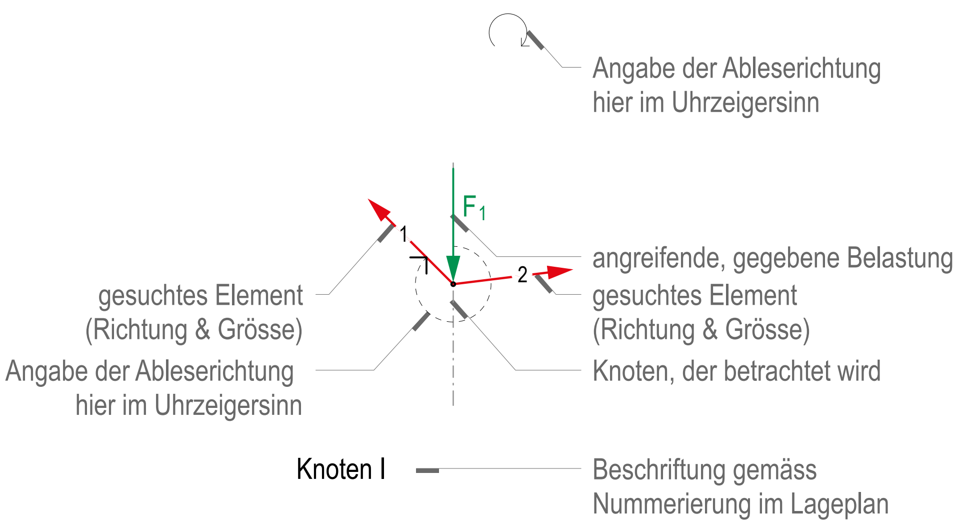
The form diagram shows the geometry of the structure with all bearing elements and the position of the loads. Loads (F_1, F_2) and reaction forces (A, B) are called „external forces“ and are drawn with their direction (as an arrow). They are drawn in the colour green. Forces in the structural elements (segments 1-3) are called internal forces and do not have a clear direction. They are coloured red for tension and blue for compression depending on the type of stress. The form diagram is drawn in a certain scale. Example: 1:100 means that 1 cm in the plan corresponds to 100 cm in reality.



Form Diagram1:100

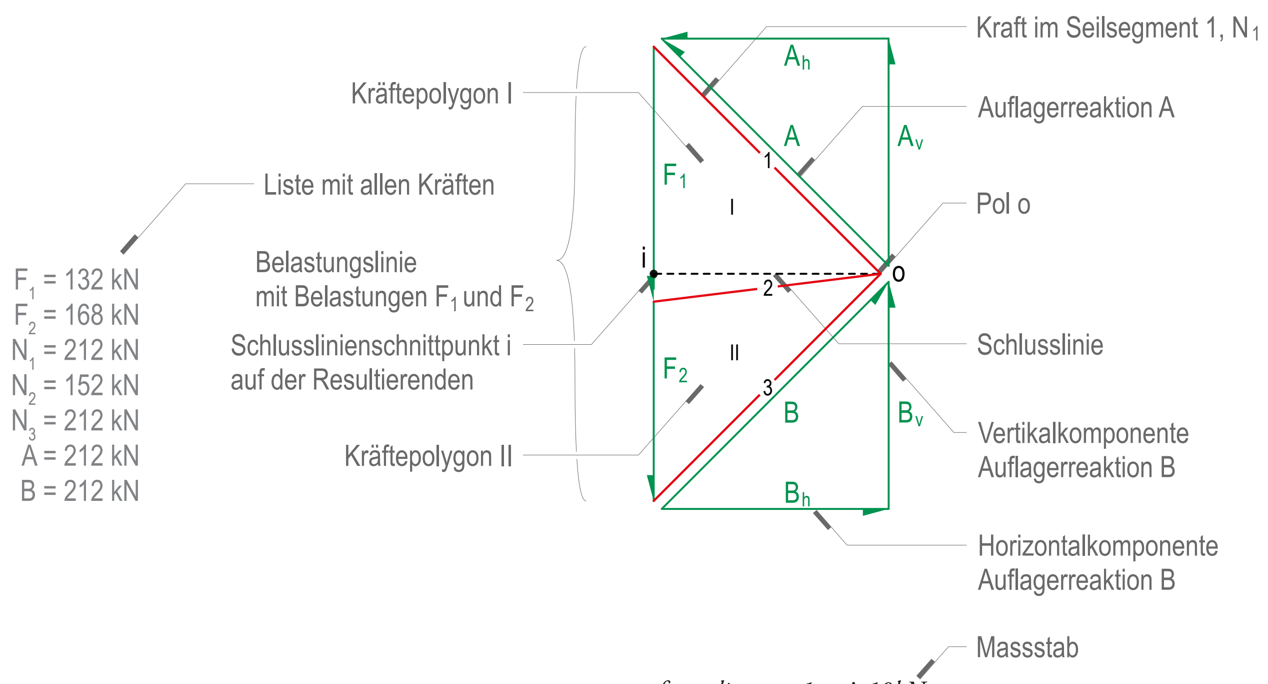
Subsystem

The subsystem (also free body diagram – FBD) and the reading direction declare the order of the elements that are to be drawn when constructing the force diagram. The subsystem is displayed without scale, usually as a sketch. In the subsystem the forces / elements acting on the nodes are drawn and a distinction is made between given and searched elements. With the subsystem and the force directions it contains, it can be determined whether the structural element is loaded in tension or compression. If the force points towards the node, it is a compression force (blue). If it points away from the node, it is a tensile force (red).



Force diagram

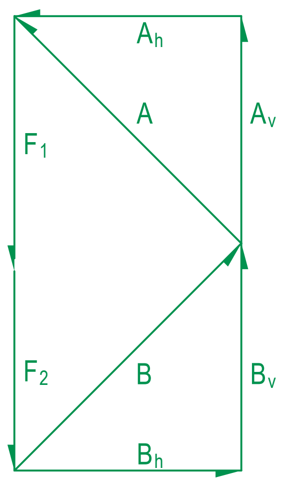
The force diagram is constructed by parallel shifting of the structural elements from the form diagram. The chosen reading direction and the corresponding subsystem indicate the drawing order of the elements. The magnitudes of the forces can then be measured directly from the complete force diagram using the given scale. 1cm ≙ 50kN means that 1cm in the plan corresponds to a force of 50 kN.



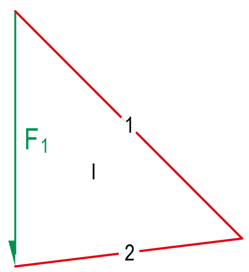
force diagram 1cm ≙ 10kN

Global and local equilibrium

The load line is the force polygon of all external forces (actions and reaction forces). If the polygon is closed, the entire system is in equilibrium (global equilibrium). A single node (subsystem) is in equilibrium if all forces acting on the node form a closed polygon in the force diagram. The force polygon is the model of the local equilibrium of the internal forces.



global equilibrium



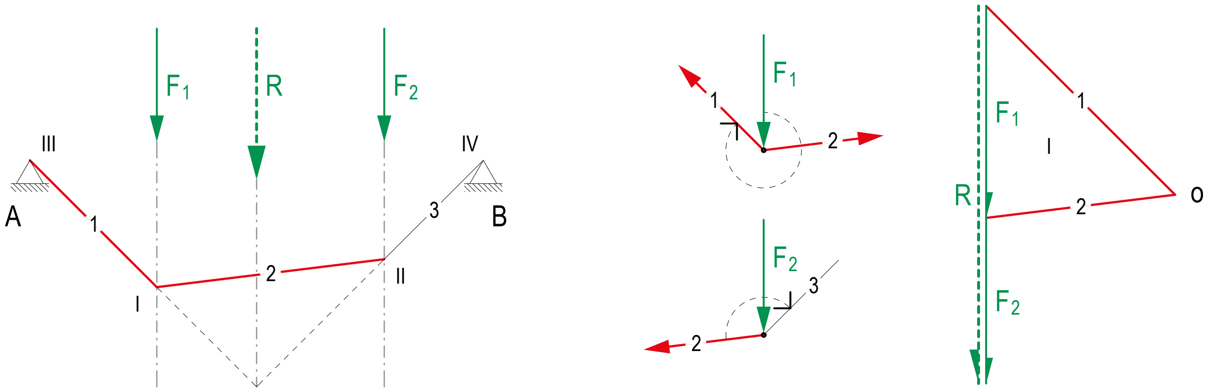
local equilibrium node I

2.2 Compendium Structural Design I&II

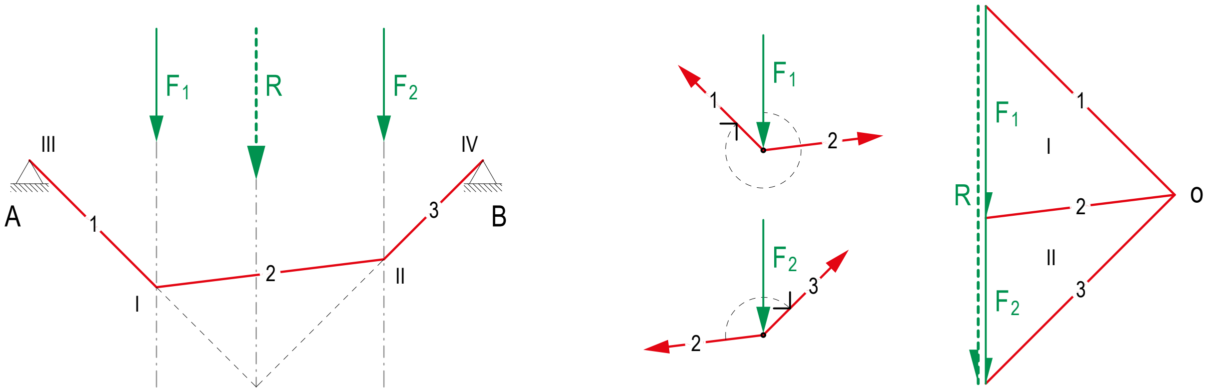
Analysis: Step by step

Given is the form of a structure between the two supports A and B. The two acting point loads F_1 and F_2 divide the cable into three segments, since there is always a change of direction where a force is applied. We are looking for the internal forces in the cable and the two reaction forces.

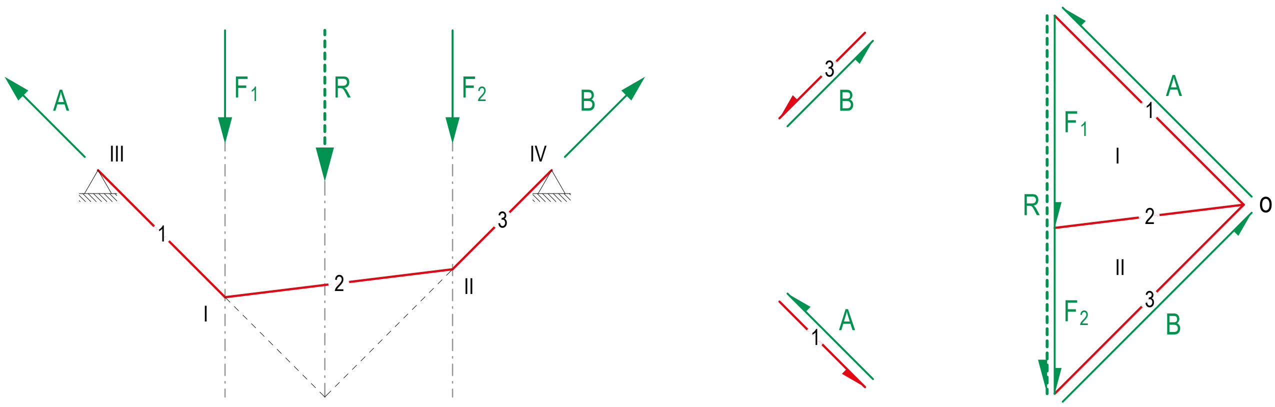
The elements in node I are transferred clockwise into the force diagram, starting with the first known force. The magnitude and the direction of F_1 are known. At the end of F_1 the first unknown element (2) is applied. Since the exact length of this element is still unknown, the second and last element (1) is placed at the beginning of F_1. F_1, together with the elements 1 and 2 form the force polygon of node I. With the help of the direction in force polygon I given by F_1, it can now also be checked whether 1 and 2 are compression or tension elements.



In node II as well, the first known force is used; in this case, this would be element 2, which is a pulling force that always acts away from the node and therefore determines the direction to the left. At the end of 2, F_2 is applied. The last unknown element of the node (3) closes the force polygon II.



Nodes III and IV are required to find the reaction forces. In node III only element 1 and the reaction force A are acting. 1 is a tensile element and A acts accordingly in the opposite direction. The magnitude of the two forces is the same. The same is true for node IV with element 3 and the reaction force B.

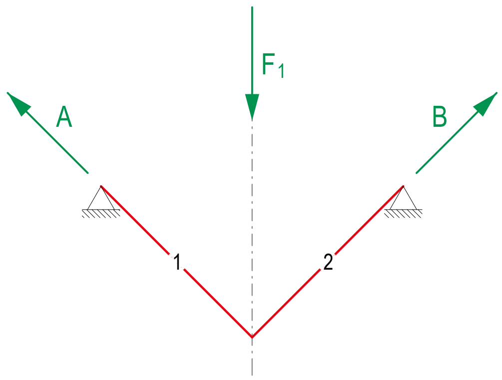


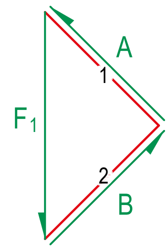
form diagrams 1:100 *force diagrams 1cm ≙ 10kN*

Comparison: Analysis – Form-Finding

Given form (Analysis)

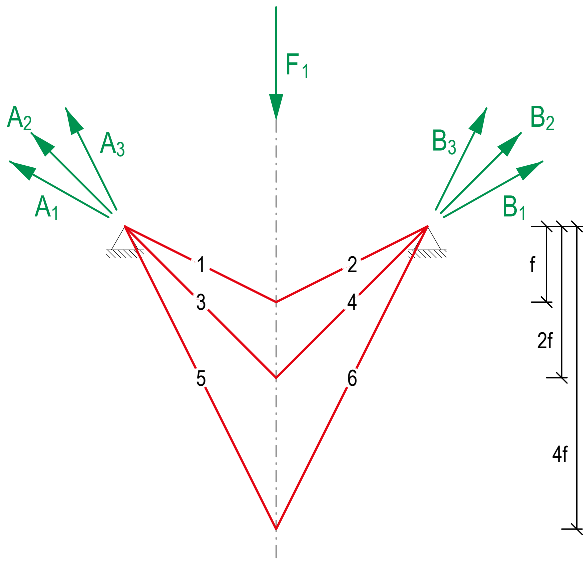
If the form of the structure is given, the forces follow the form.
After all acting forces have been drawn as load line in the force diagram, the transfer of the individual elements of the nodes begins. Only nodes with two or less unknown elements can be solved. Starting with the first known force, the elements of the node are transferred one after the other (clockwise) into the force diagram. The force diagram is completed node by node until all elements of the structure and the reaction forces are drawn at least once.



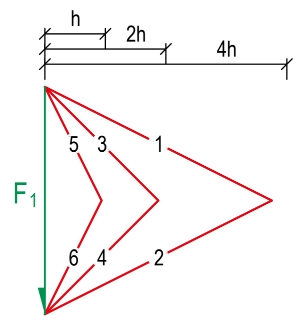


Form-finding

Often the form of the structure has to be designed. In this case the form follows the force.
First the load line is drawn. In the example below, the applied force is equal to the resultant force. The depth of the structure, the so-called rise, can now be freely selected on its line of action. The example shows three of an infinite number of possible cable structures with varying rise f.
The static depth of the structure is in relation to the magnitude of the horizontal component of the internal forces. The steeper a structure, the smaller the internal forces. If the rise is doubled, the horizontal component of the internal forces is halved, and with it the horizontal thrust of the reaction forces.



$f = \frac{1}{h}$



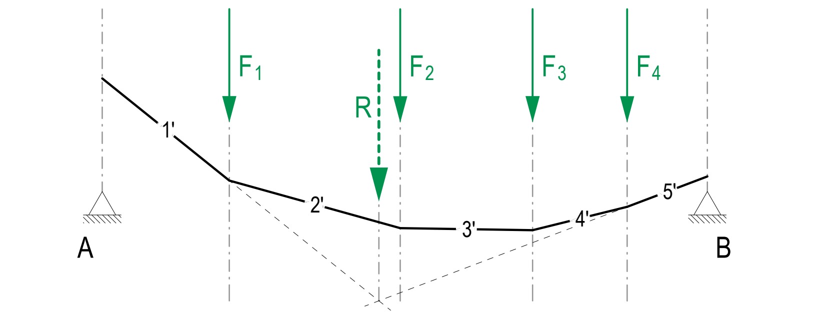
form diagrams 1:100

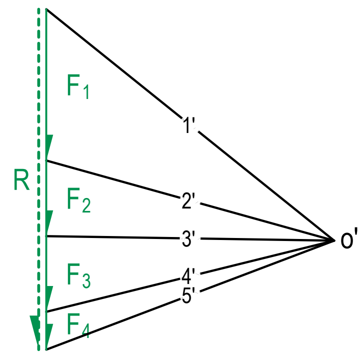
force diagrams 1cm ≙ 10kN

There are four unevenly distributed point loads and the two supports A and B. We want to find one of many possible supporting structures, which is in equilibrium under the given loading case.

1. Resultant

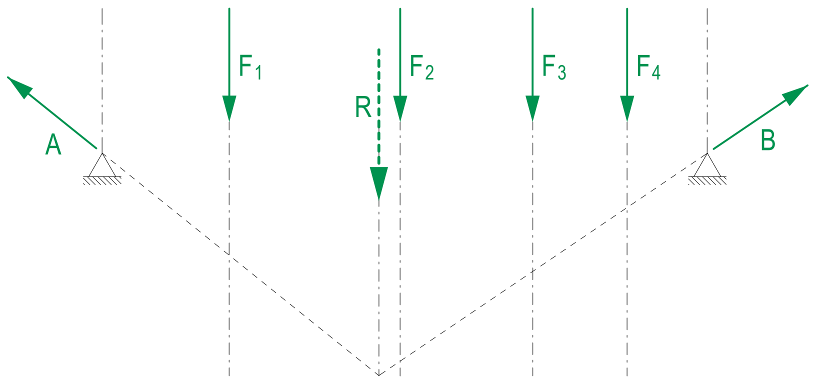
After drawing the load line, the position of the resultant in the form diagram is found by using a trial funicular.

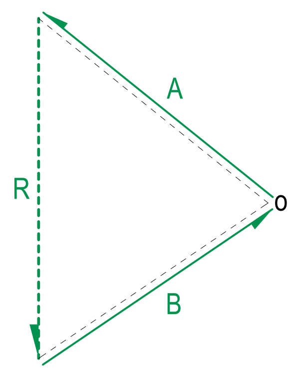




2. Global equilibrium

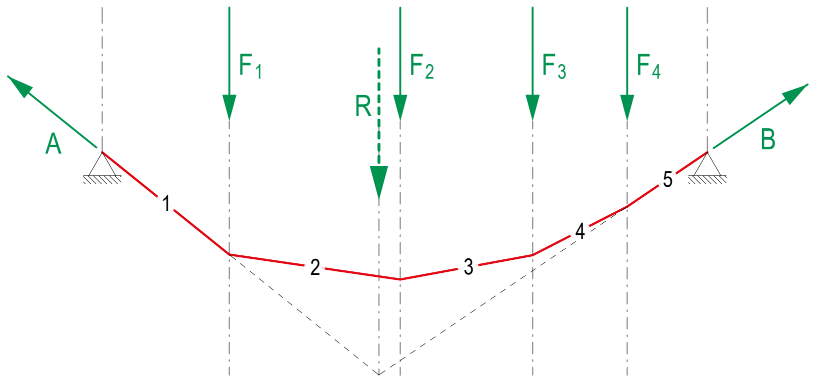
Now a point is fixed on the line of action of the resultant, and connected into the two supports. The resulting node illustrates the global equilibrium, i.e. the equilibrium between the resultant and the reaction forces.

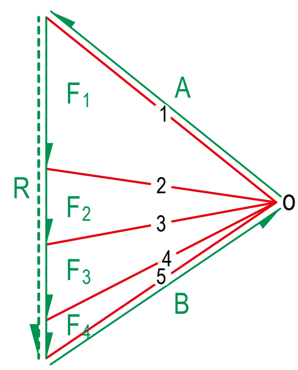




3. Local equilibrium

Starting from pole o, the rays can now be drawn in the force diagram and transferred one by one to the form diagram



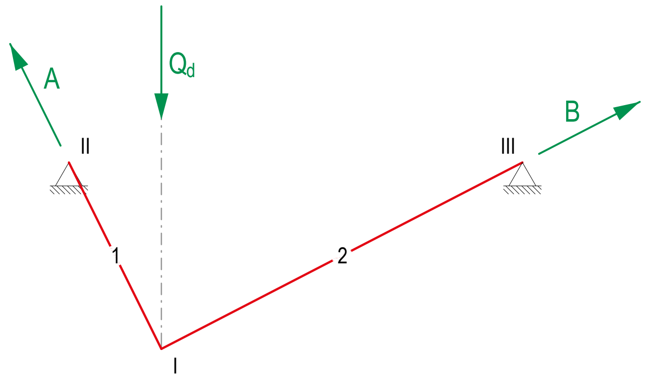


form diagrams 1:100

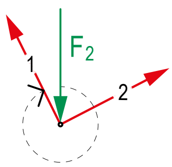
force diagrams 1cm ≙ 10kN

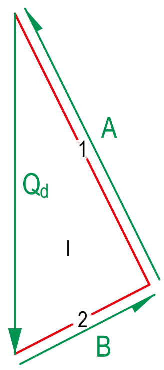
Dimensioning

Given is the form diagram of a cable made out of steel S235 under the live point load Q_k = 30 kN. The required diameter of this cable is to be found.
First the characteristic value ($_k$) of the acting force Q must be brought to the design level ($_d$). This is achieved by multiplying with the safety factor. Since the magnitude of a load over the lifetime of a structure cannot always be exactly predicted, a safety factor γ is calculated for each load. For dead loads the safety factor is γ_G=1.35 and for live loads γ_Q=1.5. With the found force Q_d the force diagram can be drawn.



$Q_d = Q_k \cdot \gamma_Q$
$= 30\ \text{kN} \cdot 1.5$
$= \underline{45\ \text{kN}}$





To calculate the cable diameter, the relevant force N_{dmax} in the structure is determined. The relevant force is understood to be the largest internal force. In this case, this is element 1 with a length of 4 cm, and therefore a magnitude of 40 kN.

$N_1 = 40\ \text{kN} = \mathbf{N_{dmax}}$
$N_2 = 20\ \text{kN}$
$A = 40\ \text{kN}$
$B = 20\ \text{kN}$

If the relevant force N_{dmax} is divided by the material strength f_d, the required cross-sectional area A_{req} is obtained.

$\mathbf{A_{req} = N_d / f_{td}}$

The strength of the given material can be taken from the formulary. Since 1 is a tensile element, the allowable tensile stress f_{tk} is relevant. A material safety factor γ_M is also included in the values of the material's strength to consider errors in the material. In contrast to the safety factor of the load, however, f_{tk} is divided by γ_M. γ_M is material-specific and can therefore also be taken from the formulary.

$f_{td} = f_{tk} / \gamma_M$
$= 235\ \text{N/mm}^2 / 1.05 = 223.81\ \text{N/mm}^2$

$A_{req} = N_d / f_{td}$
$= 40\ \text{kN} / 223.81\ \text{N/mm}^2 = 178.7\ \text{mm}^2$

Finally, the diameter is found using the formula for the circular area. Important: The result is always rounded up, as rounding off would result in a diameter smaller than the minimum requirement.

$A = r^2 \cdot \pi = (D/2)^2 \cdot \pi$
$D = 2 \cdot \sqrt{A/\pi}$
$= 2 \cdot \sqrt{178.7\ \text{mm}^2 / \pi} = 15.08\ \text{mm} \approx \underline{16\ \text{mm}}$

Stress proof

A cable cross-section of steel S355 with a diameter D=20mm under a relevant tensile force N_d = 80kN is given. The proof is sought whether the cross-section of the cable can withstand the given load.

$\mathbf{N_d \leq N_{allow} = f_{td} \cdot A_{ef}}$

First, the maximum allowed force N_{allow} of the cable is to be found. This is calculated by multiplying the designed allowable tensile stress f_{td} with the effective cross-sectional area A_{ef} based on the given diameter of the cable.

$A_{ef} = r^2 \cdot \pi = (D/2)^2 \cdot \pi$
$= (20\ \text{mm}/2)^2 \cdot \pi = 314.16\ \text{mm}^2$

Second, the found force N_{allow} is then compared with the relevant force N_d. If N_{allow} is equal to or larger than N_d, the proof is provided and the given cross-section withstands the applied load. If the proof is not fulfilled, the cable must be re-dimensioned.

$N_{allow} = f_{td} \cdot A_{ef}$
$N_{allow} = 338.1\ \text{N/mm2} \cdot 314.16\ \text{mm}^2 = \underline{106.2\ \text{kN}}$

$N_d = 80\ \text{kN}$ $\qquad \mathbf{N_d \leq N_{allow}}$

2.6 Compendium Structural Design I&II

Formulary

Bemessungsformeln / *Dimensioning Formulas*

Belastungsart / *Nature of force:*	**Bemessen / *Dimension:***	**Tragsicherheitsnachweis / *Proof:***
Zug / *Tension*	$A_{req} = \dfrac{N_d}{f_{td}}$ [mm²]	$N_d \leq N_{allow} = f_{td} \cdot A_{ef}$ [N]
Druck / *Compression* (Materialversagen / *Material failure)*	$A_{req} = \dfrac{N_d}{f_{cd}}$ [mm²]	$N_d \leq N_{allow} = f_{cd} \cdot A_{ef}$ [N]

Tragfähigkeitsformeln / *Formulas of load-bearing capacity*

Bemessungswert der Zugfestigkeit
Design value allowable tensile stress $f_{td} = f_{tk} / \gamma_M$ [N/mm²]

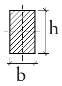
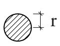
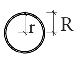
Bemessungswert der Druckfestigkeit
Design value allowable compressive stress $f_{cd} = f_{ck} / \gamma_M$ [N/mm²]

Bemessungswert der Kraft
Design value of force $F_d = F_k \cdot \gamma$ [kN]

Sicherheitsfaktoren für Lasten / *Safety factors for loads*:

Ständige Lasten / *Dead load*: $\gamma_G = 1.35$
Veränderliche Lasten / *Live load*: $\gamma_Q = 1.5$

Querschnittwerte / *Section properties*

Rechteck/ *Rectangular*		$A = b \cdot h$ [mm²]
Kreis/ *Circle*		$A = r^2 \cdot \pi$ [mm²]
Kreisring/ *Circular ring*		$A = (R^2 - r^2) \cdot \pi$ [mm²]

Formulary

Legende / *Legend*

Kräfte (innere und äussere) / *Forces*

N	Normalkraft / *Axial force*	[kN]
V	Querkraft / *Shear force*	[kN]

Lasten / *Loads*

F	Punktlast allgemein / *Point load general*	[kN]
G	Einzellast, ständig / *Dead point load*	[kN]
Q	Einzellast, veränderlich / *Live point load*	[kN]
s	Linienlast allgemein / *Line load general*	[kN/m]
g	Linienlast ständig / *Dead line load*	[kN/m]
q	Linienlast veränderlich / *Live line load*	[kN/m]
$\bar{s}$	Flächenlast allgemein / *Area load general*	[kN/m²]
$\bar{g}$	Flächenlast ständig / *Dead area load*	[kN/m²]
$\bar{q}$	Flächenlast veränderlich / *Live area load*	[kN/m²]

Geometrie / *Geometry*

A	Querschnittsfläche / *Cross-sectional area*	[mm²]
l	Länge / *Length*	[mm]
r	Radius / *Radius*	[mm]
d	Durchmesser / *Diameter*	[mm]
t	Dicke / *Thickness*	[mm]
b	Breite / *Width*	[mm]
h	Höhe / *Height*	[mm]
Δl	Längenänderung / *Length variation*	[mm]

Index / *Indices*

k	Charakteristischer Wert / *Characteristic value*
d	Wert auf Bemessungsniveau / *Design value*
q	veränderliche Last / *Live load*
g	ständige Last / *Dead load*
allow	Zulässige ... / *Allowable ...*
cr	Kritische Knicklast / *Critical buckling load*
req	erforderliche ... / *Required ...*
eff	effektive ... / *Effective ...*
t	Zug ... / *Tension ...*
c	Druck ... / *Compression ...*

Materialkennwerte / *Material properties*

Holz ***Timber***	Zugfestigkeit f_{tk} *Allowable tensile stress f_{tk}* [N/mm²]	Druckfestigkeit f_{ck} *Allowable compressive stress f_{ck}* [N/mm²]	Raumlast γ_k *Material density γ_k* [kN/m³]	Widerstandsbeiwert γ_M *Material safety factor γ_M*
Fichte *Spruce*	14	20	4.5	1.7
Buche *Beech*	24	26	6.5	
Eiche *Oak*	26	26	7.5	
BSH *Glulam*	18	22	5	

Stahl ***Steel***	Zugfestigkeit f_{tk} *Allowable tensile stress f_{tk}* [N/mm²]	Druckfestigkeit f_{ck} *Allowable compressive stress f_{ck}* [N/mm²]	Raumlast γ_k *Material density γ_k* [kN/m³]	Widerstandsbeiwert γ_M *Material safety factor γ_M*
S 235	235	235	80.0	1.05
S 355	355	355		
S 500	500	500		

Beton ***Concrete***	Zugfestigkeit $f_{tk\ (unbewehrt)}$ *Allowable tensile stress $f_{tk\ (unreinforced)}$* [N/mm²]	Druckfestigkeit f_{ck} *Allowable compressive stress f_{ck}* [N/mm²]	Raumlast γ_k *Material density γ_k* [kN/m³]	Widerstandsbeiwert γ_M *Material safety factor γ_M*
C12/15	1.1	12	25	1.5
C20/25	1.5	20		
C35/45	2.2	35		
C55/65	2.9	55		

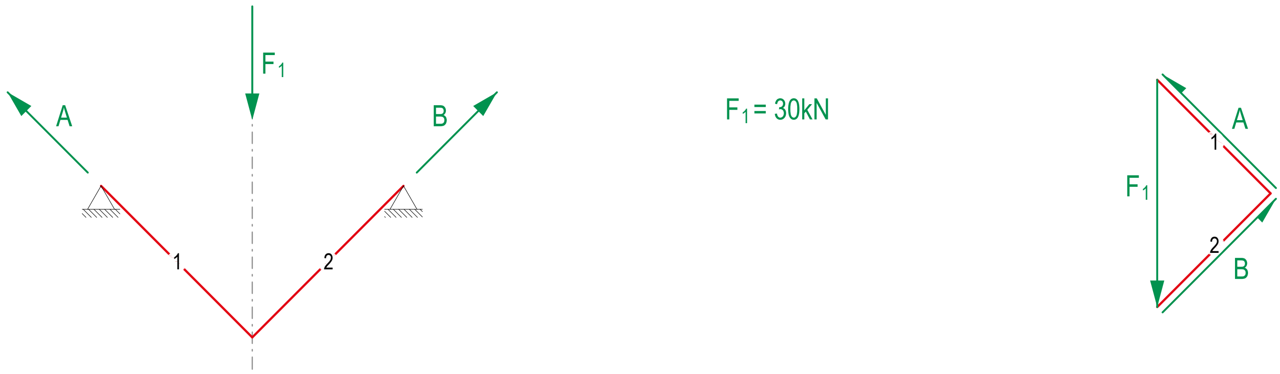
f	Materialfestigkeit / *Resistence of materials*
γ_k	Raumlast / *Material density*
γ_M	Widerstandsbeiwert / *Material safety factor*

3.1

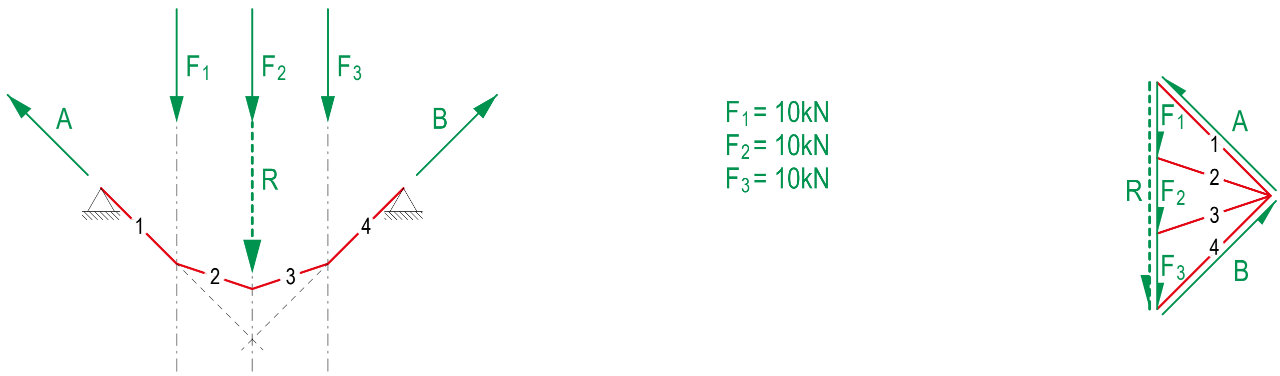
Compendium Structural Design I&II

From Point Loads to Uniformly Distributed Loads

An acting force causes a change of direction in the structure. Therefore in the example below a node with the acting force F_1 and the two rope segments 1 and 2 is created.



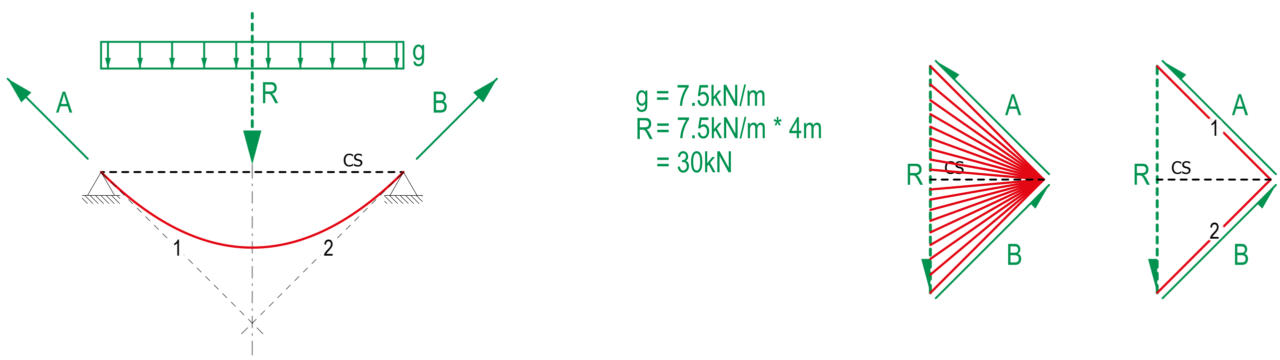
Three acting forces form a cable with four segments. There is always a change of direction where each of the forces act on the structure.



If infinitely many point loads act on the system, this is called *uniformly distributed load* or *line load*. The applied line load means a continuous application of force over a certain length and thus causes a continuous change of direction in the structure. Under a uniformly distributed load, a curve (parabola) is formed.

To find the resultant of a line load, its magnitude [kN/m] is multiplied by its length [m]. Once the resultant has been calculated, it can be drawn in the force diagram as usual. In the form diagram it is always in the centre of the applied line load.

As each change of direction leads to an additional member in the force diagram, there would have to be an infinite number of members in the case of the parabola. To simplify, only the outermost segments, i.e. the tangents to the curve at the supports, are transferred to the force diagram because that is where the inner force is largest.

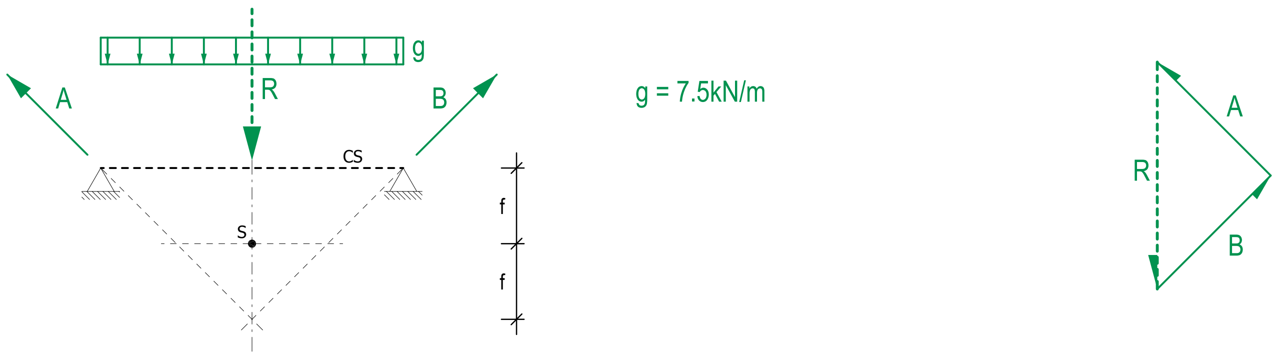


form diagrams 1:100 *force diagrams 1cm ≙ 10kN*

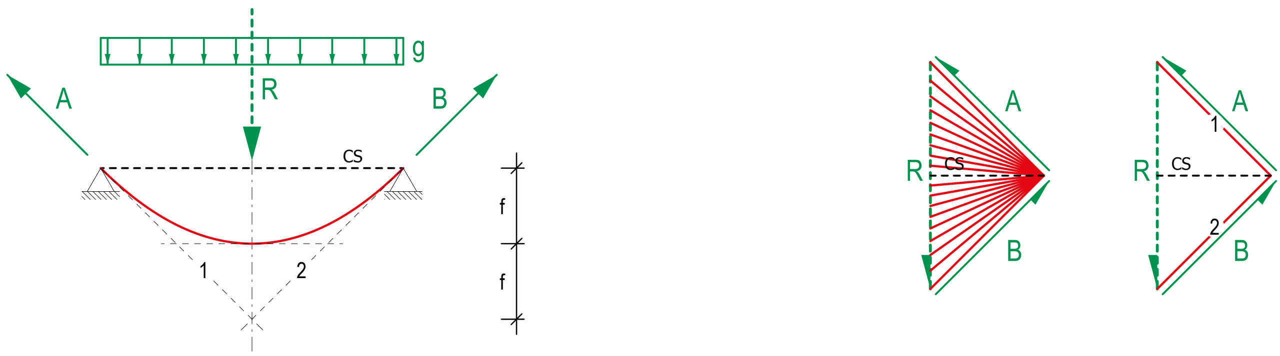
Parabola

Under a uniformly distributed load, a parabola forms. This is constructed as follows:

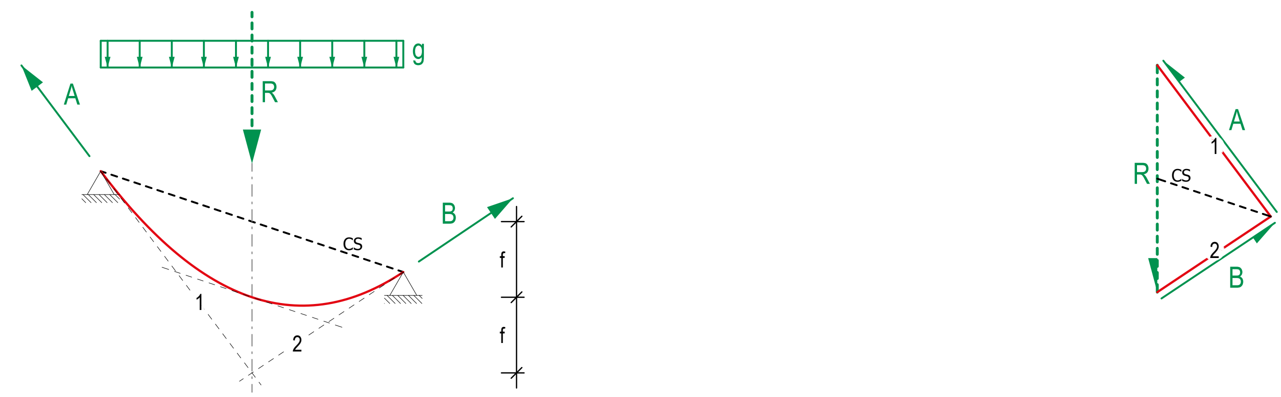
First the resultant is calculated and the global equilibrium is drawn in the force diagram. Then the outermost tangents to the curve are determined; their magnitude and inclination correspond to the reaction forces. Both elements intersect on the line of action of the resultant. The distance from the tangent intersection point to the connection of the two supports (so-called *closing string*) corresponds to twice the height f of the parabola. As a third tangent to the curve, the closing string is shifted parallel through the vertex S of the parabola.



With the visual help of the three tangents the curve can be drawn into the form diagram by hand.
In the force diagram only the outermost tangents are shown, since the forces in the structure are largest directly at the supports.



In case of an asymmetrical position of the supports, the resultant is calculated again and the global equilibrium is drawn in the force diagram. It is important that the closing string is shifted parallel along the line of action of the resultant up to the vertex of the parabola. Then the curve can be drawn into the form diagram by hand by following the three tangents.



form diagrams 1:100

force diagrams 1cm ≙ 10kN

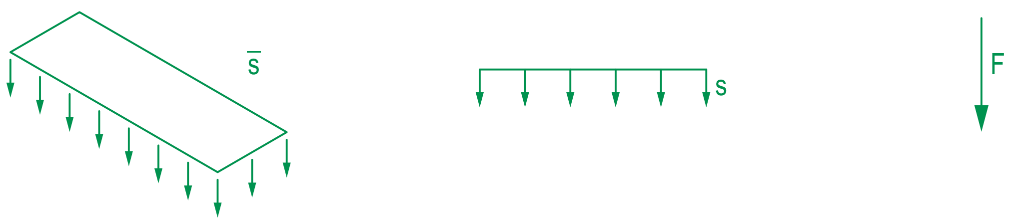
Loads

We distinguish between three different types of load: the point load, the line load and the area load. In order to recognise from the designation which type of load it is, point loads are indicated with capital letters, line loads with lower case letters and area loads with lower case letters with a line on top.

area load $\bar{s}$
unit: [kN/m^2]

line load s
unit: [kN/m]

point load F
unit: [kN]

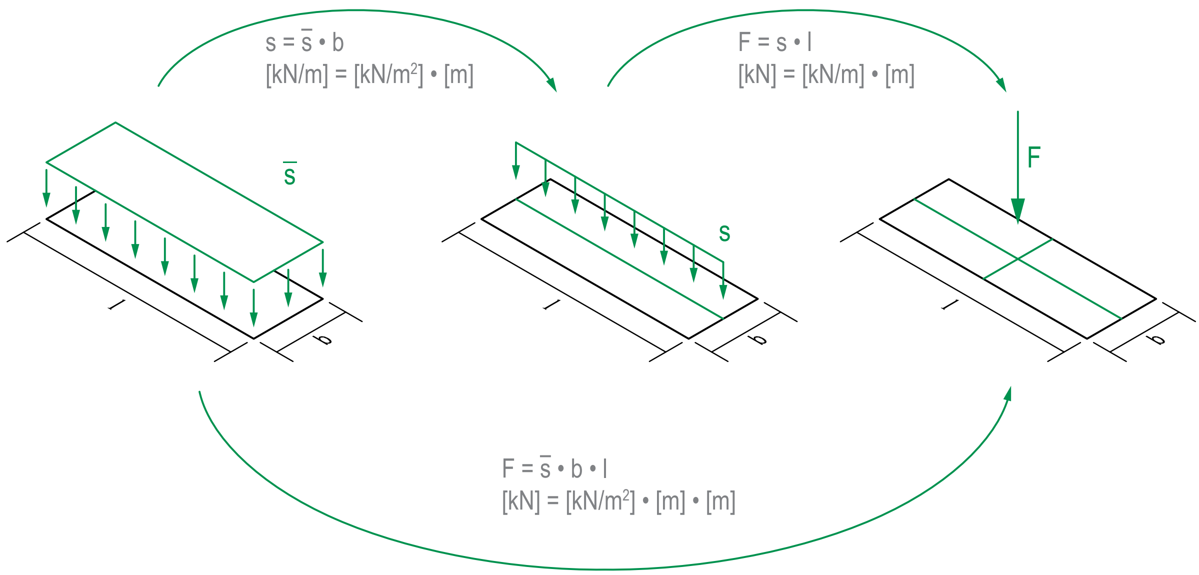


Conversion of different types of loads

Most forces result from area loads, such as the weight of a roof covering or a layer of snow, which act on the vertical elements over the entire area of a system. If the uniformly distributed load on a linear component of the structure, is to be determined, the area load is multiplied by the width of the tributary area. This results in the unit kN/m, i.e. the force per linear metre on the specifically considered linear element. This situation can be abstracted further by multiplying by the length of the tributary area, i.e. the length of the element. The result is a resulting point load which is applied centrally over the element.

To convert the area load $\bar{s}$ to a line load s which has the same length l, we need to multiply $\bar{s}$ with width b.

To convert the line load s to a point load F, we need to multiply s with the length l.



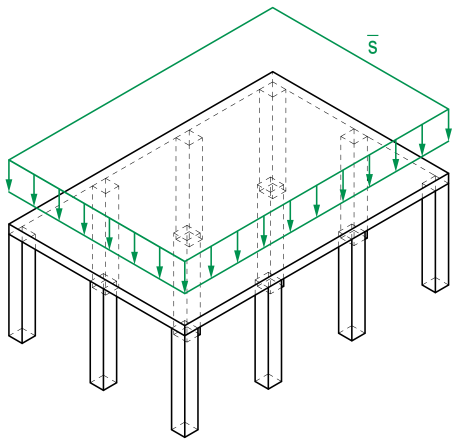
To convert area load $\bar{s}$ to a point load F, we need to multiply $\bar{s}$ with the width b and length l.

3.4 Compendium Structural Design I&II
Tributary Areas

A surface load acts on the entire structure. To determine the load that a relevant vertical element must carry, the so-called tributary area is determined. This is a partial area of the area load. As a general rule, loads directly go to the nearest support. Therefore, the distance between two load-bearing elements is halved in each case to find the dimensions of the respective tributary area.

Tributary area of a column

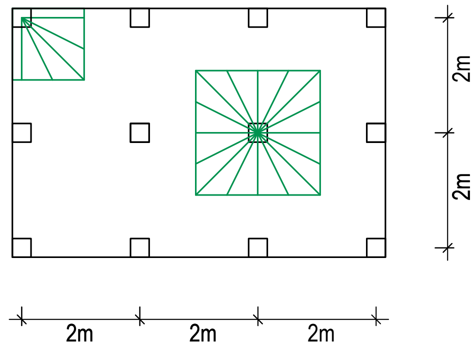
A load situation is given with an applied surface load acting on a slab, which in turn rests on a series of supports. If the distances between the supports are halved, the respective tributary areas of the supports are created. In the following example, the tributary area on a central column is four times as large as that on a corner column. Therefore, when dimensioning, the centre column is considered, as it experiences the largest force. To calculate the resulting point load on the centre column, the area load is multiplied by the size of the tributary area.



$A = 2\ \text{m} \cdot 2\ \text{m} = 4\ \text{m}^2$

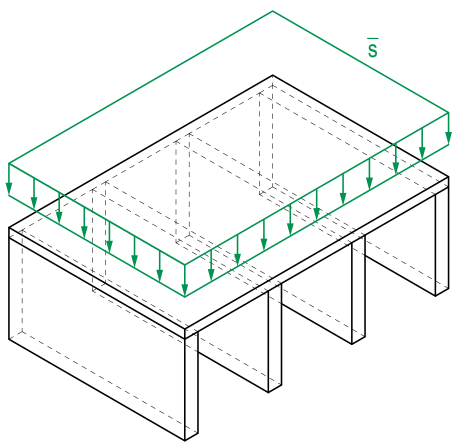
$\bar{s}_d = 1\ \text{kN/m}^2$

$R = \bar{s}_d \cdot A = 4\ \text{kN}$



Tributary area of a beam

A load situation is given with an applied surface load acting on a plate, which in turn rests on a series of beams. Analogous to the example above, the respective tributary areas are created when the distances between the beams are halved. In the following example, the tributary area of a middle beam is twice as large as that on a beam at the edge of the slab. When dimensioning, therefore, the middle beam is again considered, as it experiences the largest force. In order to calculate the resulting point load, the area load is multiplied by the size of the tributary area. Since the wall is a linear element, the magnitude of the line load might also be of use. This is calculated by dividing the resultant by the length of the element.

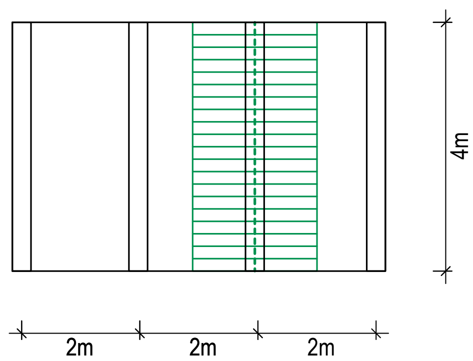


$A = 2\ \text{m} \cdot 4\ \text{m} = 8\ \text{m}^2$

$\bar{s}_d = 1\ \text{kN/m}^2$

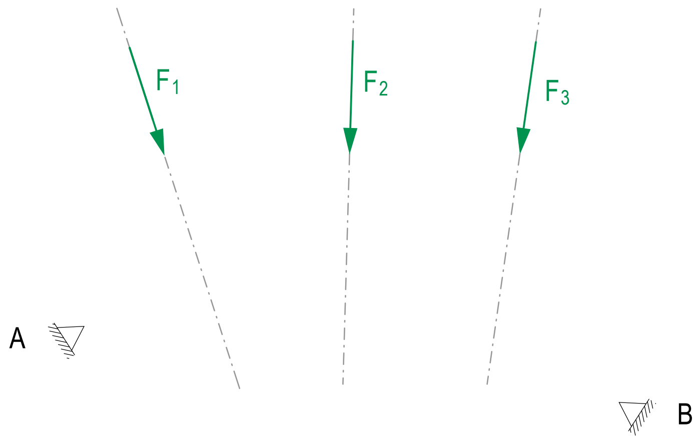
$R = \bar{s} \cdot A = 8\ \text{kN}$

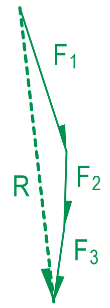
$g_d = R / l = 8\ \text{kN} / 4\ \text{m} = 2\ \text{kN/m}$



Point Load

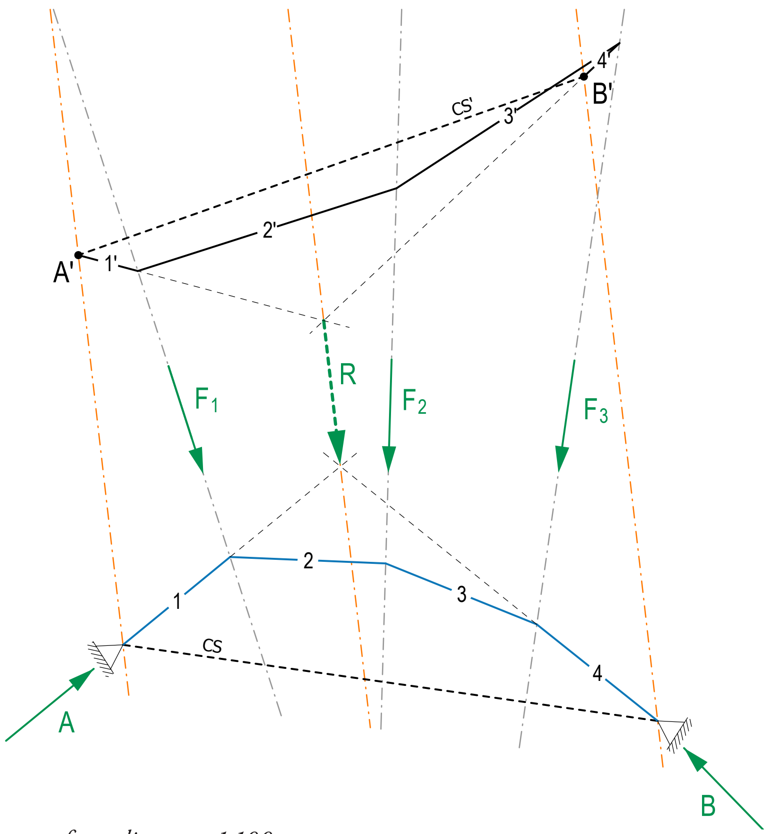
There are three non-concurrent point loads and the two asymmetrically placed supports A and B. One of many possible supporting structures, in equilibrium under the given load, is sought.

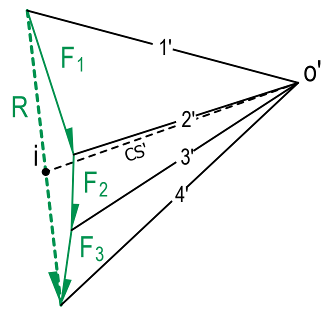


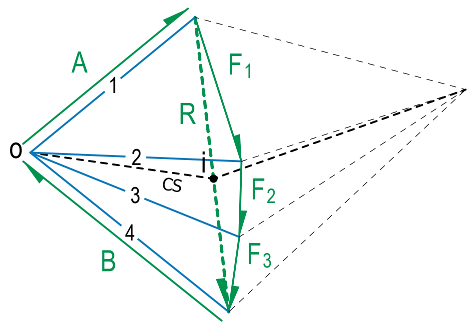


The inclination of the resultant found with the help of the load line is shifted in parallel through the two supports in the form diagram. Now the trial funicular is constructed starting from the line of action through support A and ending on the line of action through support B. The closing string CS‘ between points A‘ and B‘ is then moved parallel through the pole o‘ in the force diagram.

The intersection point i of CS‘ with the resultant in the force diagram is constant, independent of the shape of the trial funicular polygon. The point i is therefore also called the intersection point of closing strings.
Therefore, the closing string CS, i.e. the connection between A and B, is also shifted parallel through this point i. In the force diagram, any point on the closing string can now be selected as pole o, provided that no further conditions are given. The corresponding elements 1 to 4 are then transferred to the form diagram and result in a thrust line through the supports A and B.







form diagrams 1:100

force diagrams 1cm ≙ 10kN

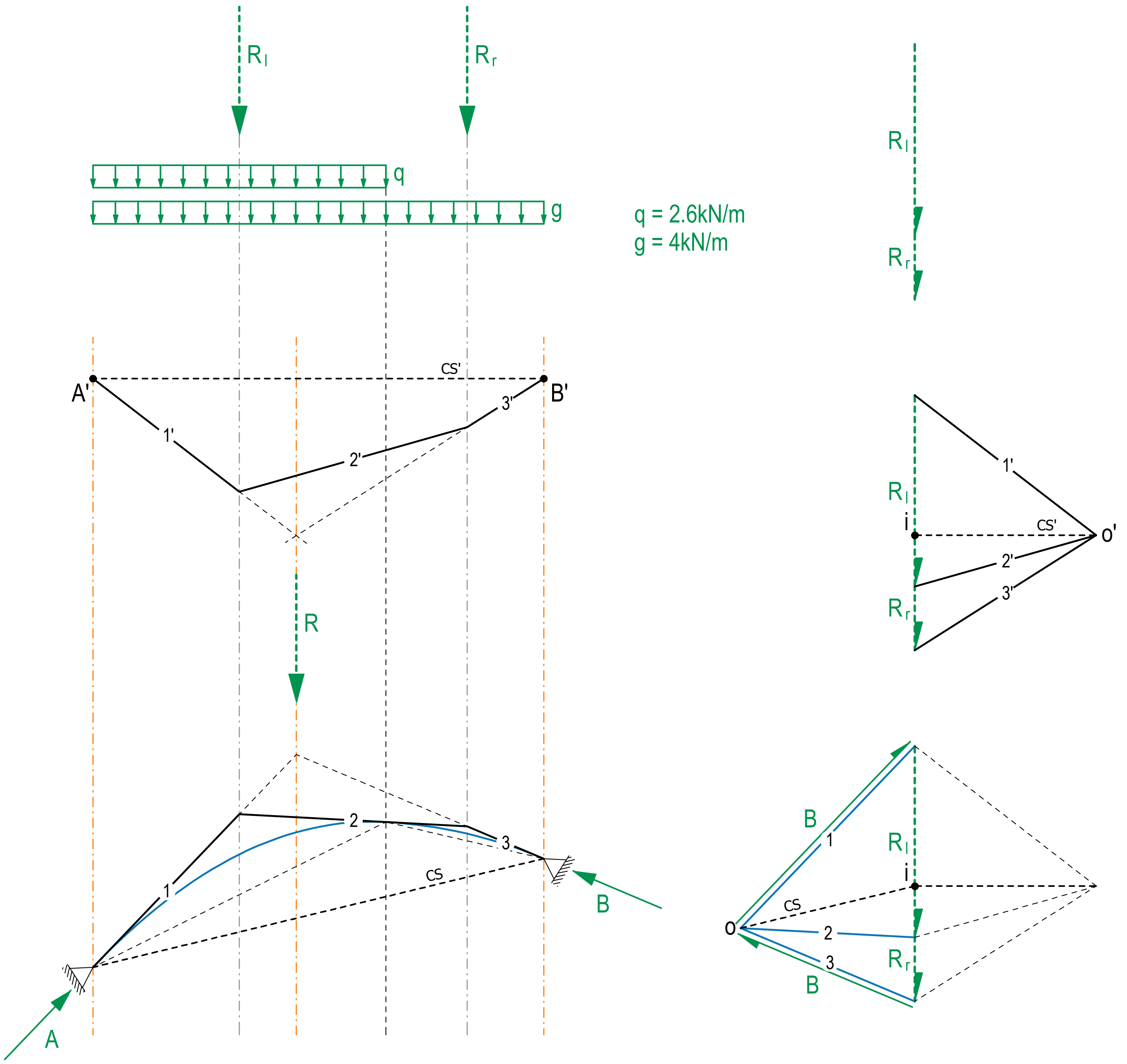
Thrust line: Trial funicular

Line Loads

There are two non-uniformly distributed loads and the two asymmetrically placed supports A and B. One of many possible supporting structures, in equilibrium under the given load, is sought.
First, the load is divided into a left and a right subsystem and the respective resultants are calculated.

The line of action of the resultant is shifted parallel through the two supports. Now, any pole o‘ can be selected in the force diagram in order to construct a trial funicular, starting from the line of action through support A. The closing string CS‘ between points A‘ and B‘ is shifted in parallel through pole o‘ into the force diagram.

Again, the closing string CS is shifted parallel through point i and a pole o is selected. The corresponding elements 1 to 3 can now be transferred to the form diagram. These result in a thrust line through the supports A and B, which takes up the resultants R_l and R_r. The rays are at the same time the tangents of the two curves. Thus, the parabola construction can be used to determine the exact shape in the left as well as in the right subsystem.



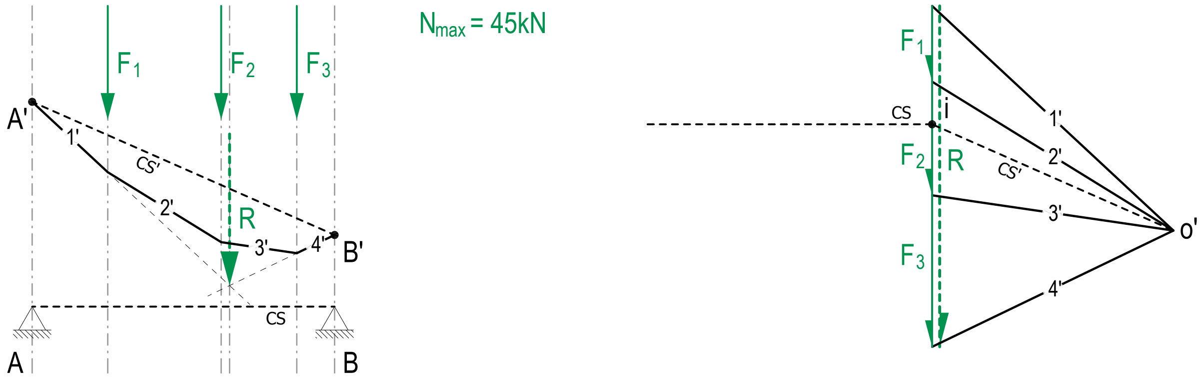
form diagrams 1:100 *force diagrams 1cm ≙ 10kN*

4.2 Compendium Structural Design I&II

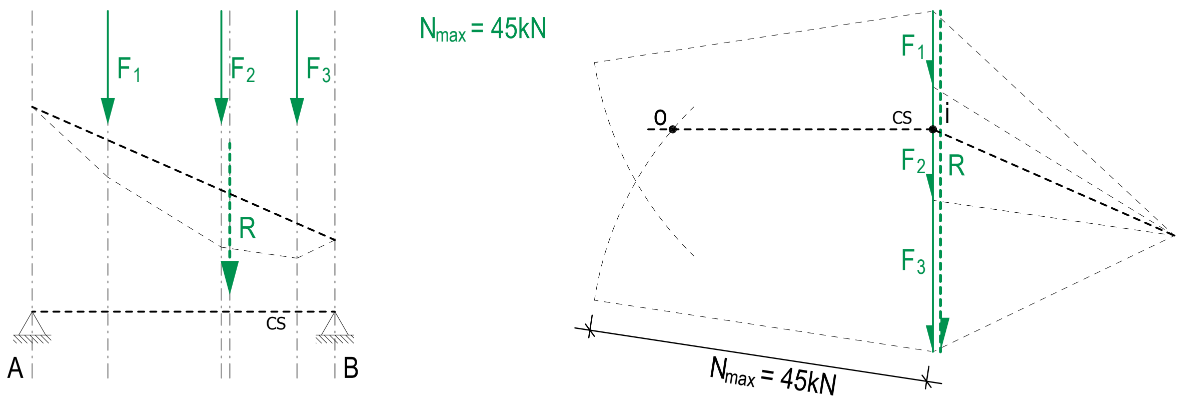
Form-finding under specific constraints

Three acting forces are given. The form of an arch in which the maximum force does not exceed 45kN is sought.

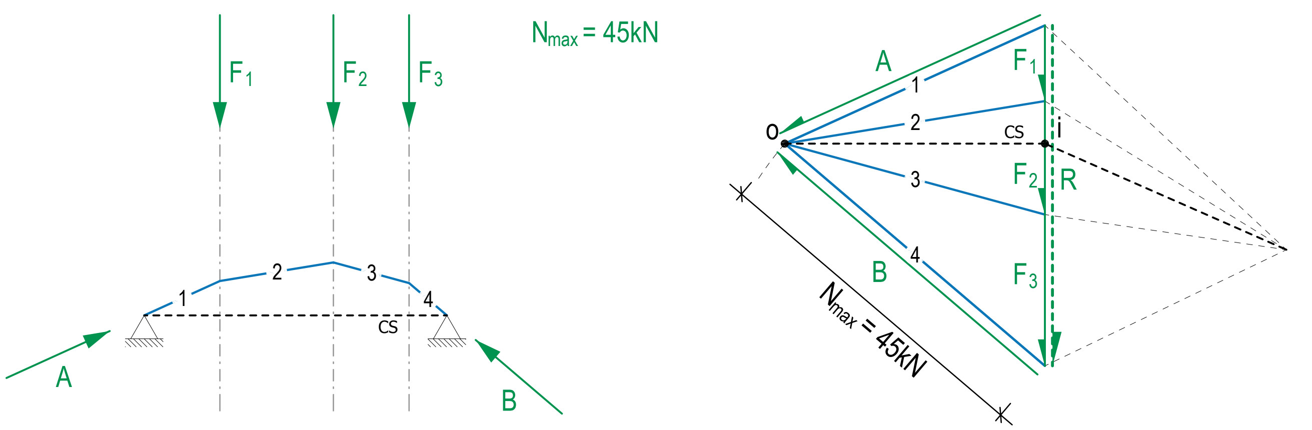
The position of the resultant and the auxiliary supports A‘ and B‘ are localized by means of a trial funicular construction. The closing string of the trial funicular CS‘ is shifted through the pole o‘ and intersects with the resultant at point i. The closing string CS also runs through i.



Since the forces are largest at the supports, the global equilibrium is now to be determined. The magnitude of the force is given, but not its direction. Therefore, the given 45kN are measured off with a compass from the beginning and the end of the resultant. Of the two resulting points of intersection with the closing string, the one closer to i is relevant - this is the pole o.



Finally, the rays can be transferred one after the other to the form diagram. The drawn arch has the minimum structural depth, which fulfils the requirement $N_{dmax} \leq 45kN$. With the specification of a maximum internal force, steeper arches are also possible, since the internal forces decrease with increasing structural depth. For other possible solutions, the pole o is therefore closer to i.

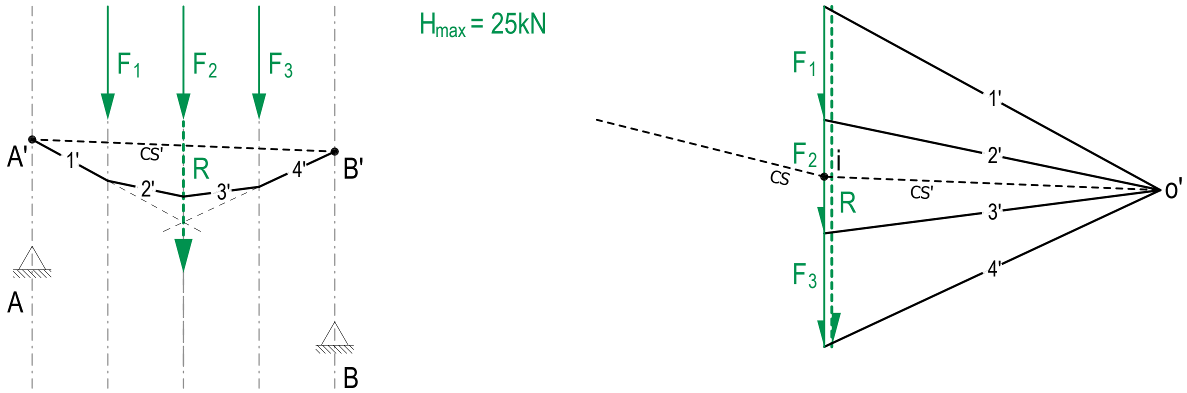


form diagrams 1:100

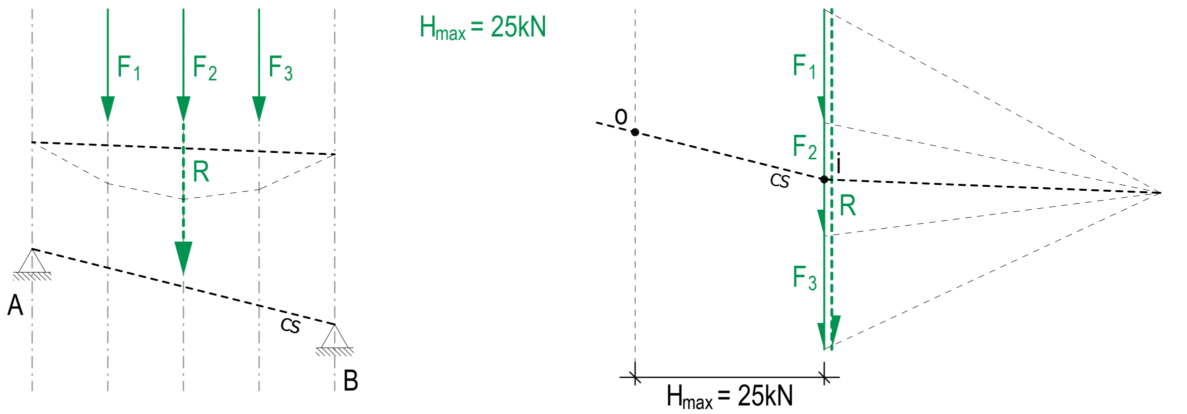
force diagrams 1cm ≙ 10kN

Three acting forces and asymmetrically positioned supports is given. The form of an arch in which the horizontal thrust in the supports does not exceed 25kN is sought.

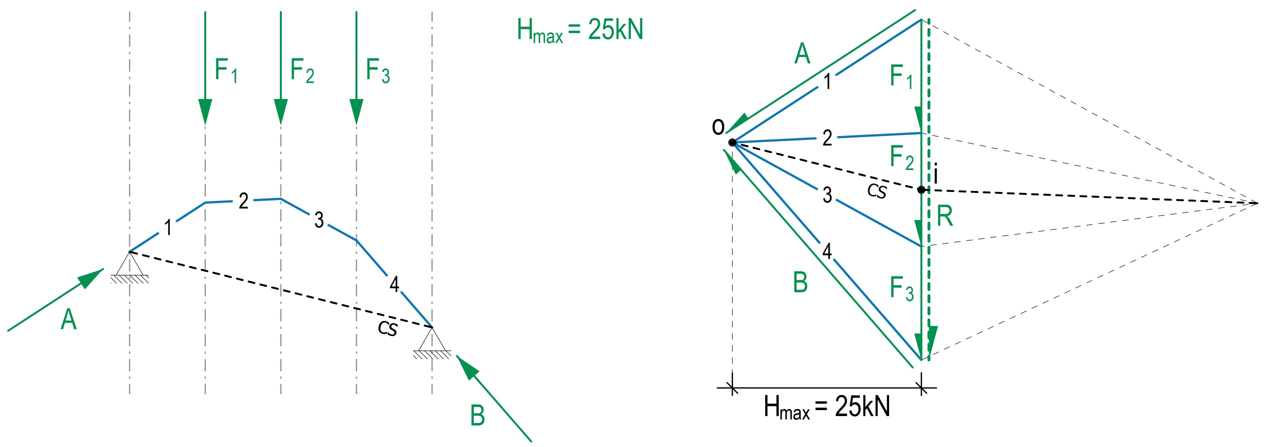
The position of the resultant and the auxiliary supports A‘ and B‘ are localized by means of a trial funicular construction. The closing string of the trial funicular CS‘ is shifted through the pole o‘ and intersects with the resultant at point i. The closing string CS also runs through i.



Now the maximum thrust H_{max} is transferred into the force diagram. For this purpose, a line is drawn parallel to the resultant, marking the maximum thrust in the supports. The pole is located at the intersection of this line with the closing string CS.



Finally, the rays can be transferred one after the other to the form diagram. The drawn arch has the minimum structural depth, which meets the requirement $H_{max} \leq$ 25kN. With the specification of a maximum horizontal component, steeper arches are also possible, since the thrust decreases with increasing structural depth. For further possible solutions the pole o is therefore closer to i.



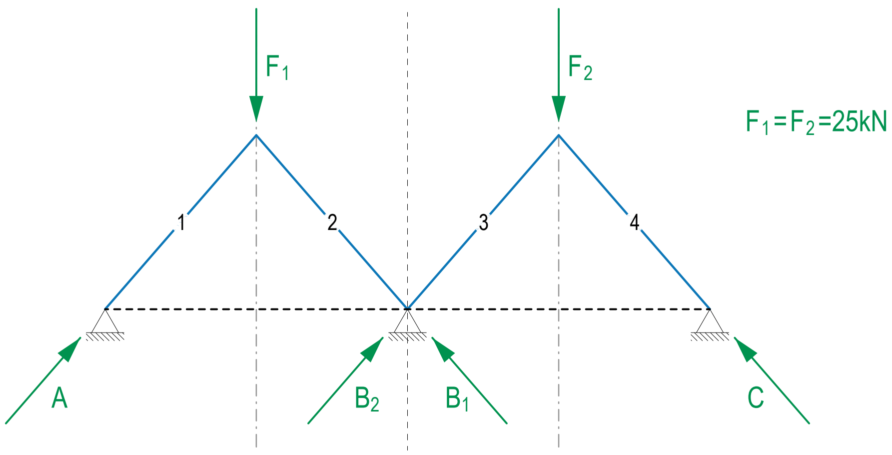
form diagrams 1:100

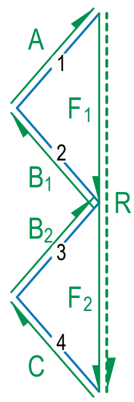
force diagrams 1cm ≙ 10kN

4.3 Dividing systems

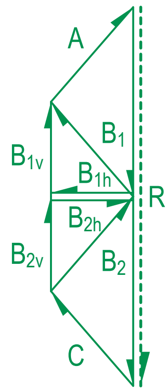
In the following a structure with three supports is analysed. Two point loads and the three supports A, B and C are given. A possible thrust line under the given load and the reaction forces are sought.

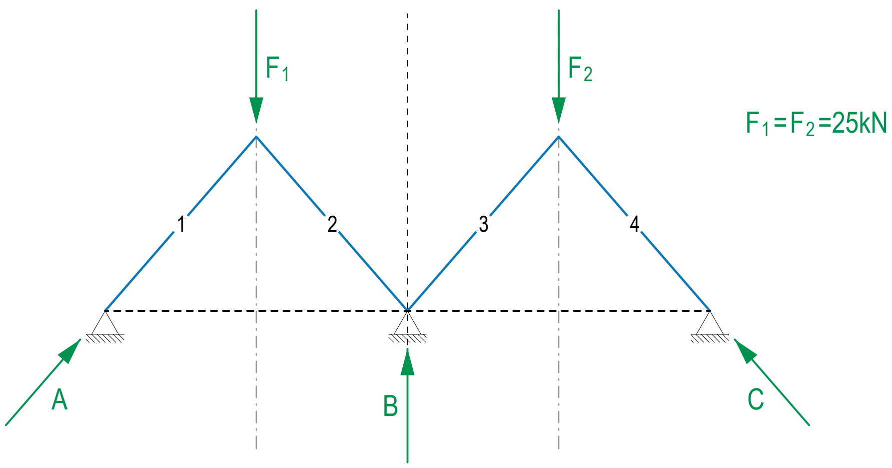
First, the system is divided into a left and a right subsystem. F_1, A and B form the left and F_2, B and C the right part. On the line of action of F_1 a point is now selected and connected to the two supports. This thrust line corresponds to the direction of the reaction forces A and B_1, which are shifted parallel in the force diagram. Equally this happens with the right subsystem, resulting in the force polygon F_2 - C - B_2.

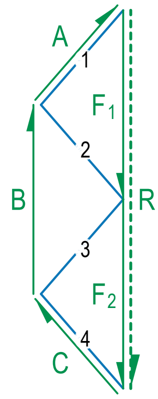




The reaction forces B_1 and B_2 can now be added up to the total reaction force B in the force diagram. Since the horizontal components of B_1 and B_2 are equal but point in opposite directions, they cancel each other out. As a result the reaction force B becomes vertical. Together with the load line, the reaction forces also result in a closed polygon, which proves that the system is in global equilibrium.







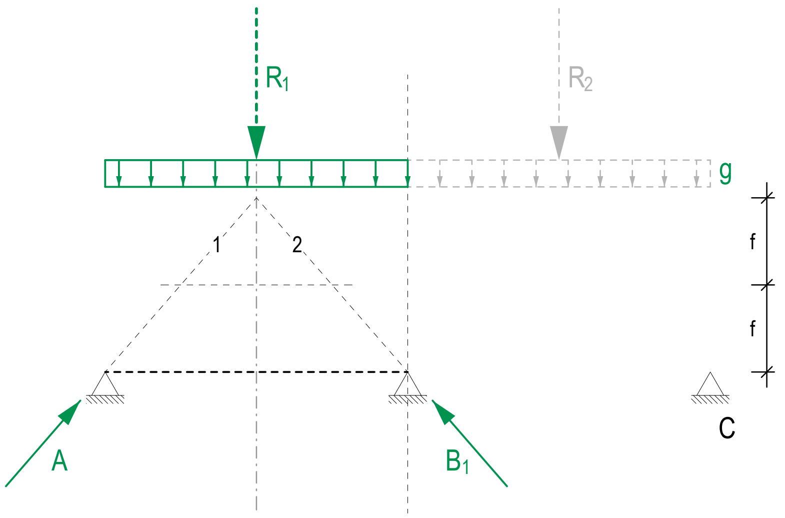
form diagrams 1:100

force diagrams 1cm ≙ 10kN

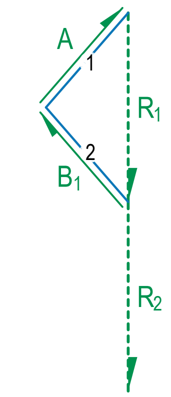
Dividing systems

In the following a structure with three supports is analysed. There is a dead line load and three supports A, B and C. A possible thrust line under the given load and the forces in the three supports are sought.

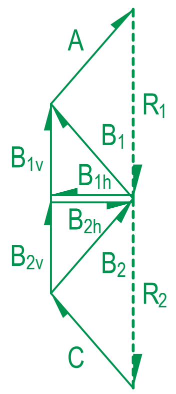
First, the system is divided into a left and a right subsystem. The line load is divided at the middle support and the two partial resultants R_1 and R_2 are calculated. A point is now selected on the line of action of R_1 and connected to the two supports. These are the tangents to the left parabola, which correspond to the direction of the reaction forces A and B_1. Equally this is done with the right subsystem.

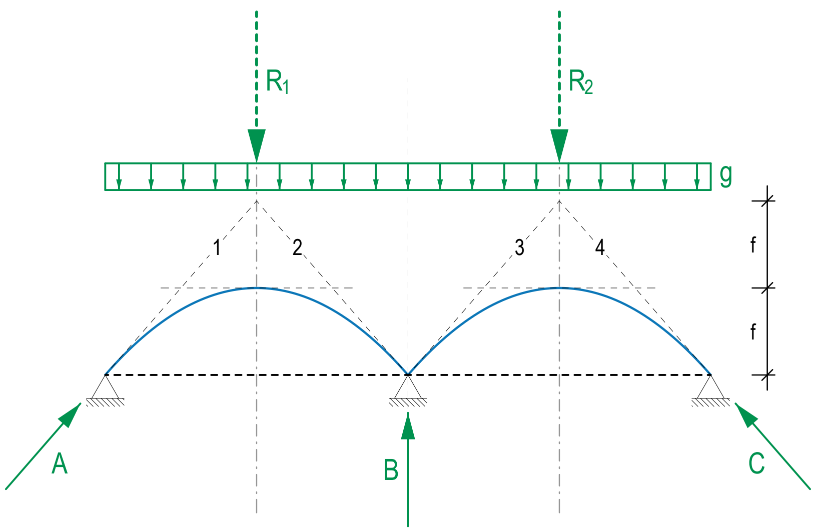


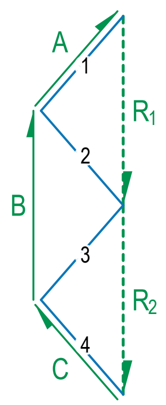
g = 6.25kN/m



Again, B_1 and B_2 can be added up to the reaction force B in the force diagram. Together with the load line, the reaction forces result in a closed polygon, which proves that the system is in global equilibrium.
Finally, the curve can be drawn into the form diagram by hand using the three tangents.







form diagrams 1:100

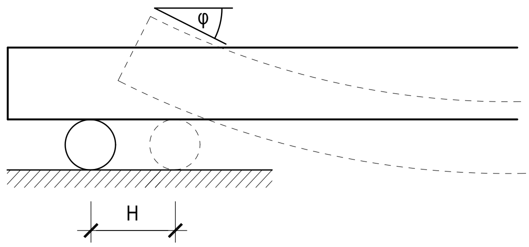
force diagrams 1cm ≙ 10kN

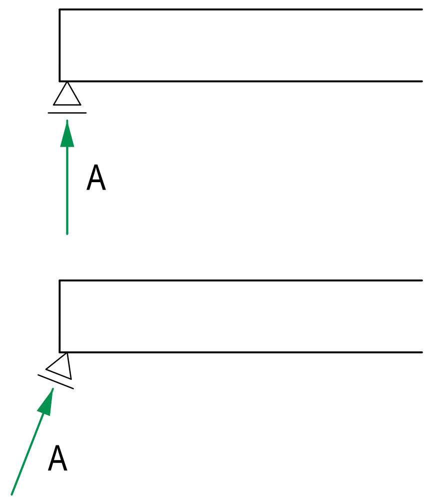
Supports

The supports are defined as the parts of structure where it is supported by other structures or by the building ground. Depending on the type of support, different types of reaction forces are possible. These depend on the different number of possible movements of the structure at the support.

Roller

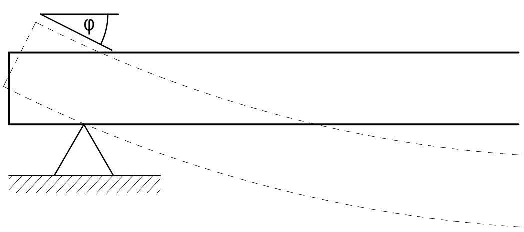
A roller allows both rotation (φ) and a horizontal movement H of the support. A roller can only take a reaction force A which is perpendicular to the shifting direction of the support. In general it is directed vertically. The roller is also called *simple support.*

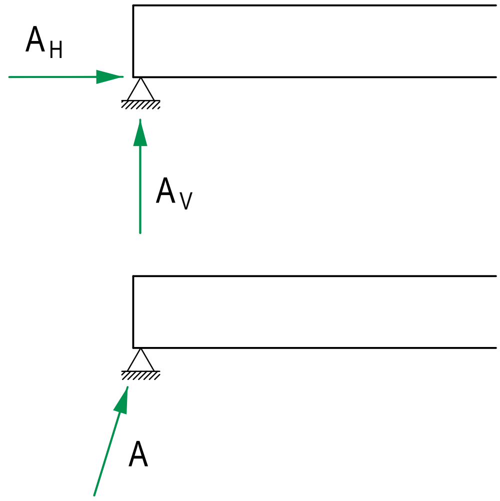




Hinge or Pin

Hinges or pins allow rotation (φ), but no displacement, horizontal or vertical movement of the support. The reaction force can be divided into the two components A_h and A_v . The hinge is also called *fixed support.*





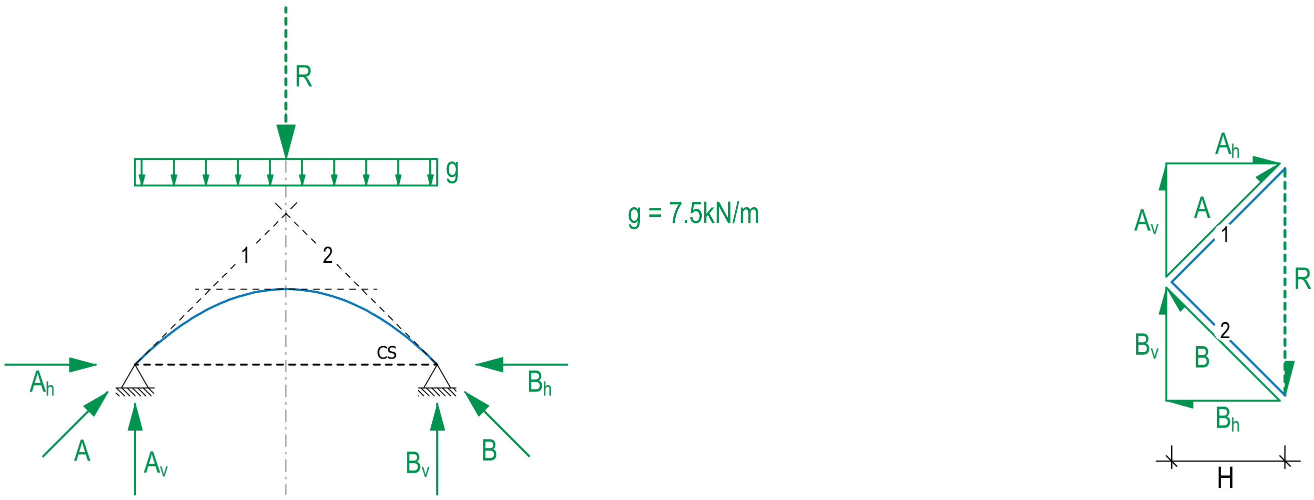
form diagrams 1:100

force diagrams 1cm ≙ 10kN

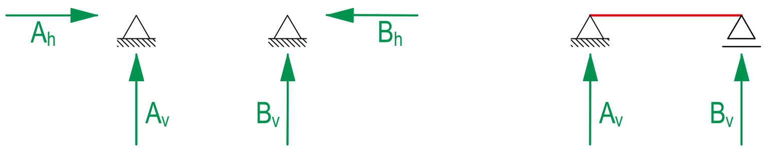
5.2 Compendium Structural Design I&II

Arch-Cable Structures

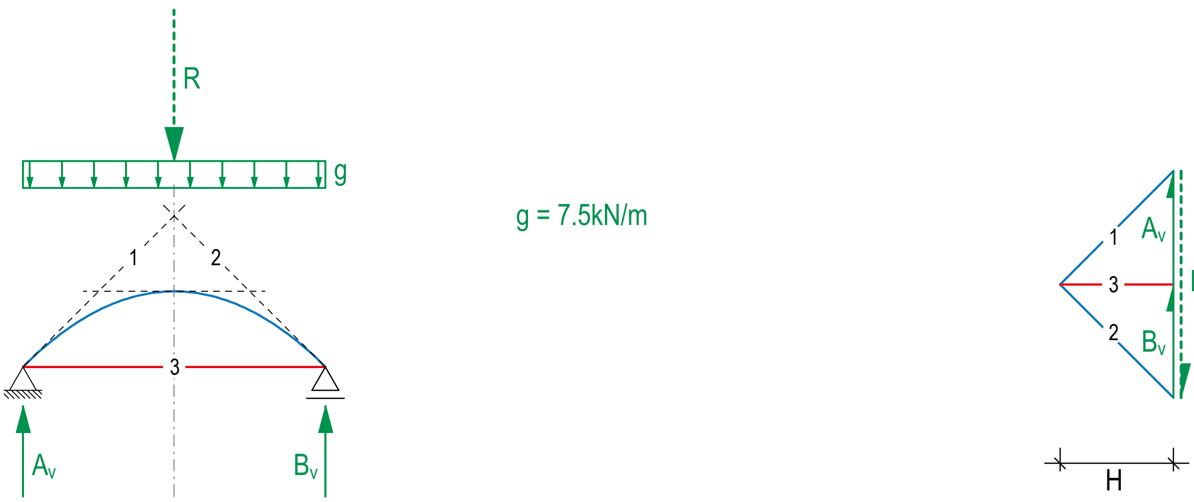
In the following, a simple arch structure, which transfers a constant line load into the two fixed supports A and B is analysed. The reaction forces can be divided into their horizontal and vertical components. A_h and B_h absorb the horizontal thrust caused by the arch, depending on its height.



Often, however, a roller, which can only absorb vertical forces is used in a structure. Therefore the horizontal thrust must be absorbed within the structure. For this purpose a tension element is inserted between the two supports. Thus both reaction forces become vertical.



Instead of a horizontal reaction force acting on the system from the outside, the internal force now pulls the arch together and prevents support B from „rolling away" to the side. The force in the cable corresponds to the horizontal components of the reaction forces. The combination of a compression-stressed arch and a tension-stressed cable is called arch-cable structure.



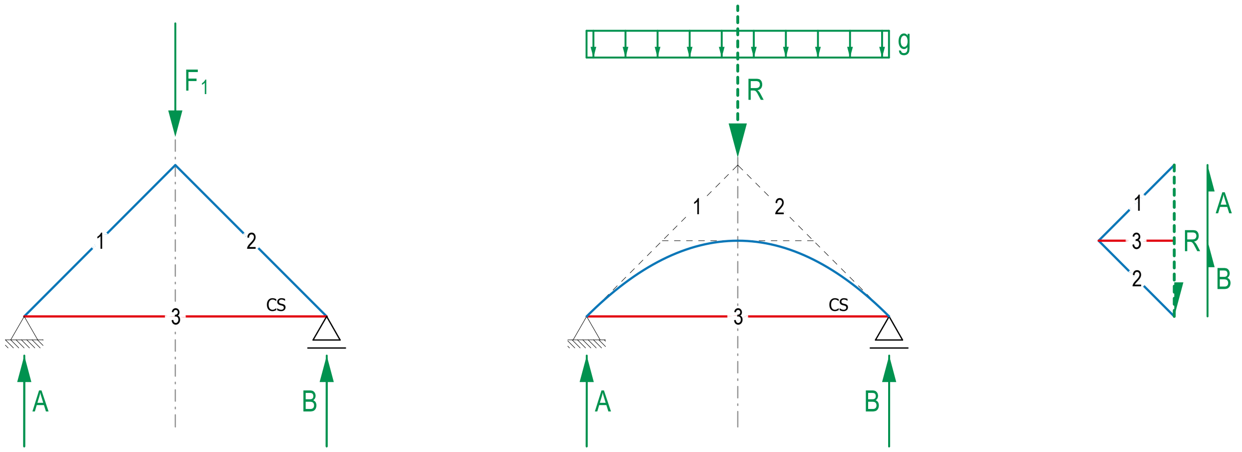
form diagrams 1:100

force diagrams 1cm ≙ 10kN

5.3 Comparison: Span and Cantilever

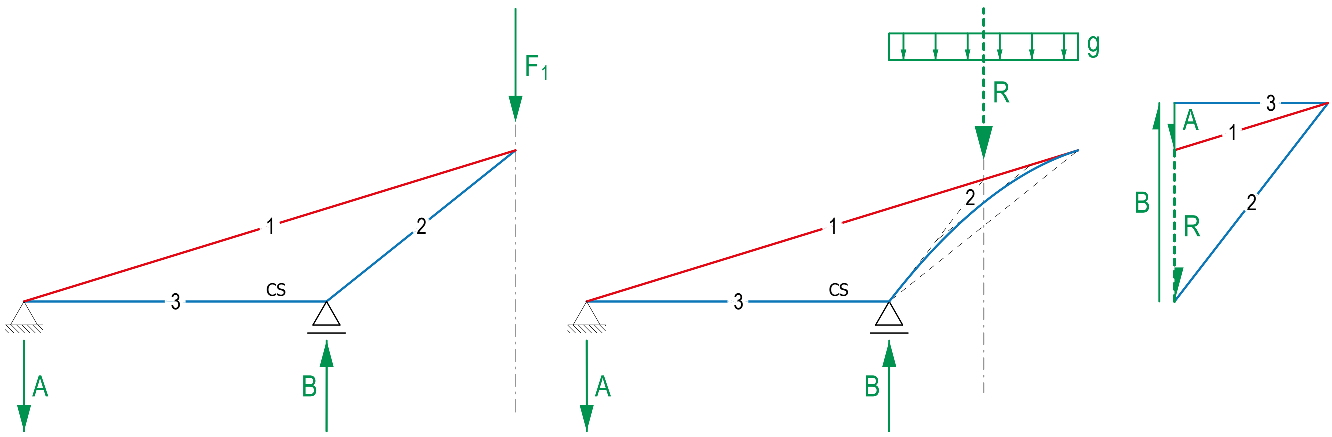
Span

If the resultant of all acting forces is located between two supports, this is a so-called *span*. Both reaction forces act in opposite direction to the applied forces.



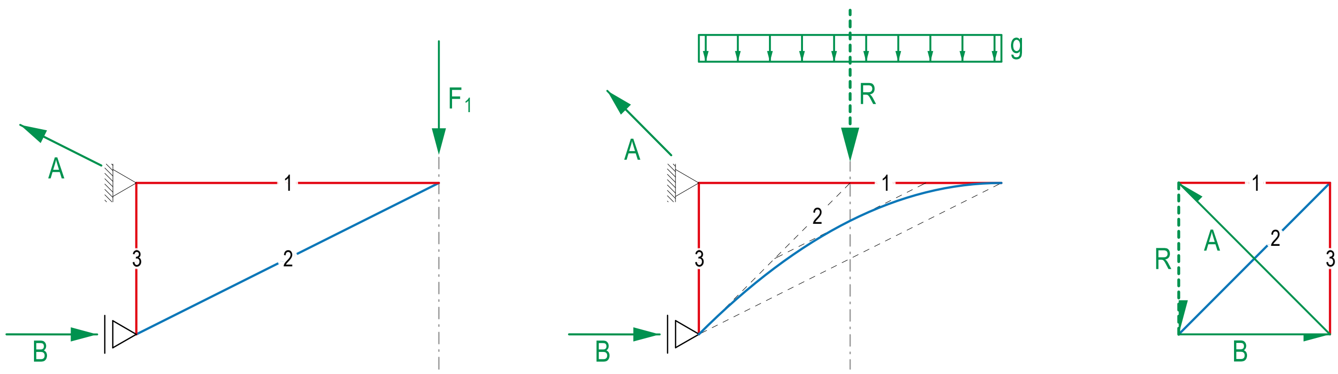
Cantilever

If the resultant of all acting forces is outside the supports, this is a *cantilever*. The reaction force A in this example is a tensile force, because otherwise the whole system would topple over (see 1.4 Stability).



Cantilever: Console

A special case of the cantilever is the console. Here, the supports are located vertically one above the other. While a horizontal arrangement of the supports also allows combinations between spans and cantilevers, this is a full cantilever only.



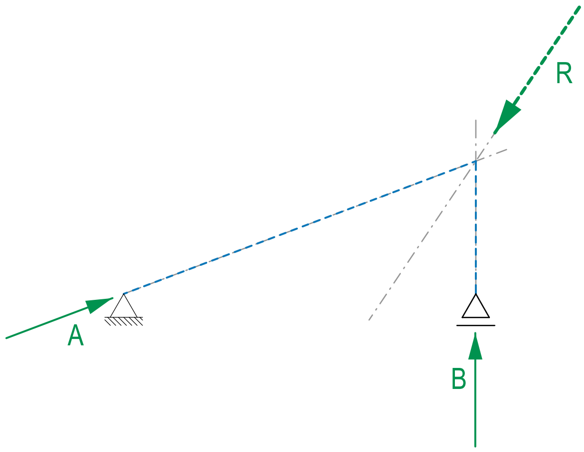
form diagrams 1:100

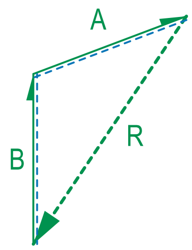
force diagrams 1cm ≙ 10kN

Reactions in the case of non parallel lines of action

In the case that the line of action of the resultant and the line of action of the reaction forces are not parallel, then they must intersect in the form diagram at one point. With this point, the global equilibrium can be determined directly graphically.

In the given example, the perpendicular line of action of support B intersects with the resultant at one point. This intersection point is connected to the support A, what gives us the direction and herefor the line of action of the support force A.

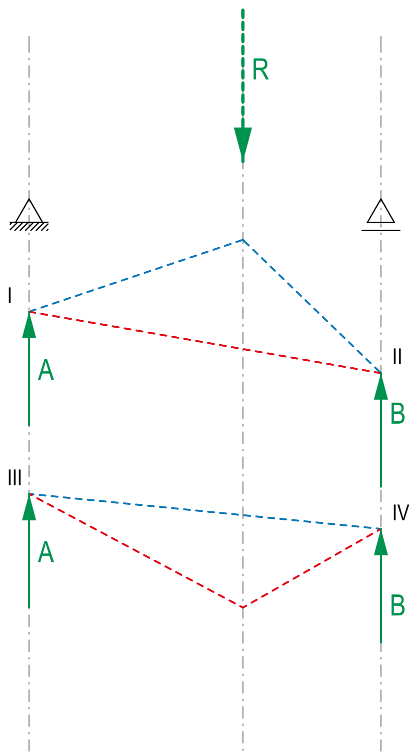


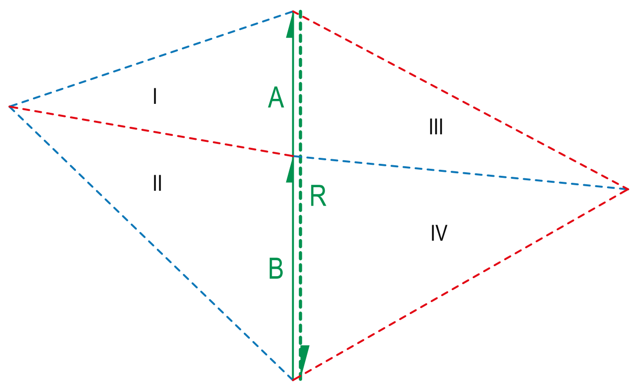


Reactions in the case of parallel lines of action

If the line of action of the resultant is parallel to the lines of action of the reaction forces, the global equilibrium and the reaction forces can be determined by bringing the applied load directly to the supports . For this purpose, any triangular structure that transfers the resultant to the supports can be chosen. Subsequently, the corresponding force diagram is drawn. The magnitude and direction of the support forces can be determined from the equilibrium of the individual nodes.

As the force diagrams show, both trial structures lead to the same solution for the magnitude and direction of the support forces.





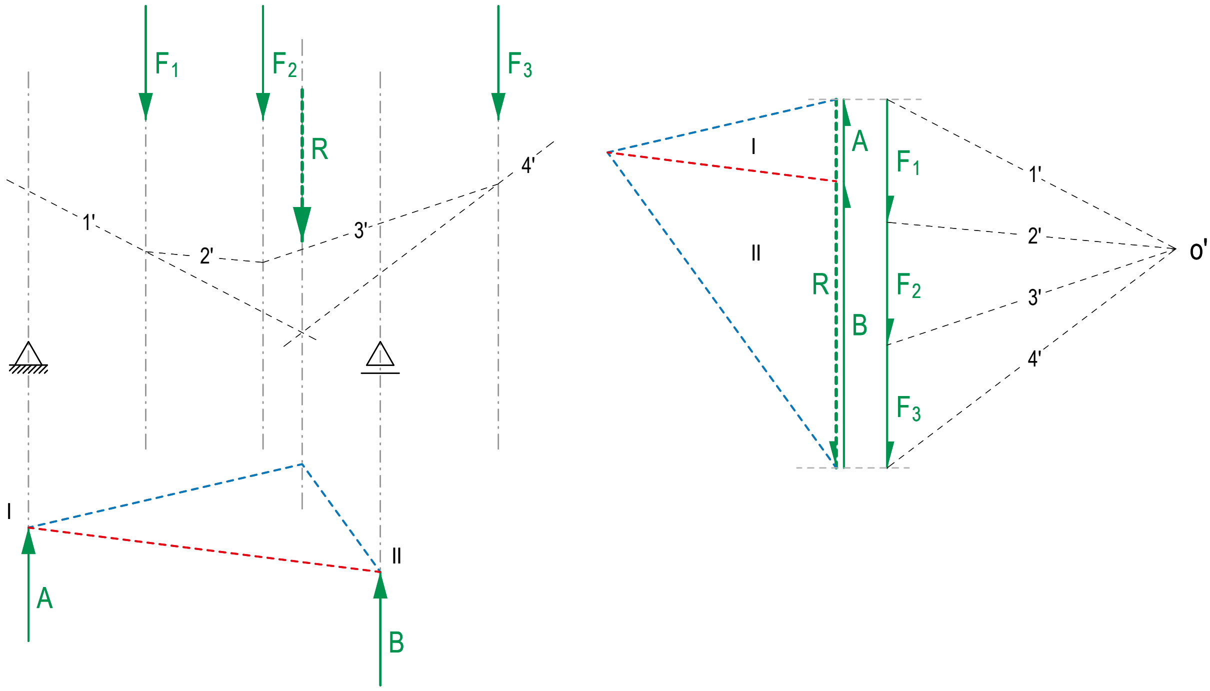
form diagrams 1:100

force diagrams 1cm ≙ 10kN

Resultant and reactions in the case of parallel lines of action

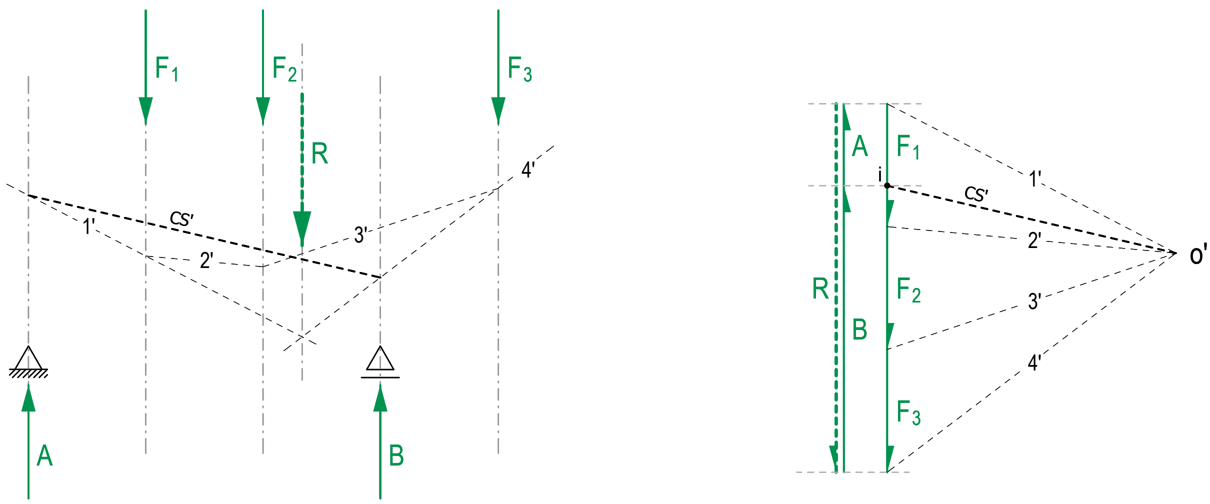
If multiple loads are being applied, the position and magnitude of the resultant must be found using a trial funicular (see 1.3) before the global equilibrium and the magnitude of the reaction forces can be determined.

Once the position and direction of the resultant are known, the magnitude of the reaction forces can be determined using a triangular structure as shown in the previous example.



Another method to determine the magnitude of the reaction forces is by using the closing of the trial funicular (CS`) string an its corresponding point i. However, this method is then suitable when the line of action of the resultant is parallel to the line of action of the reaction forces.

First, the load line is drawn in the force diagram. Then a pole o` is free chosen and the line segments are drawn to all beginnings and ends of the applied forces. These are then transferred in parallel to the form diagram. The closing string CS` found in this way is finally transferred parallel through o` into the force diagram. The point i lays in the intersection of the CS` and the load line. The point i divides the load line into the two reaction forces A and B.



form diagrams 1:100

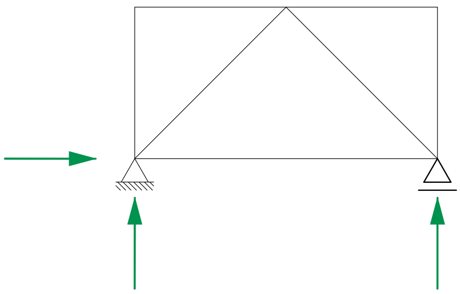
force diagrams 1cm ≙ 10kN

Internal statical Determinacy

A plane is internally statically determined if the number of truss members (S) plus the possible reaction forces (A) of the system equal twice the number of truss nodes (K).

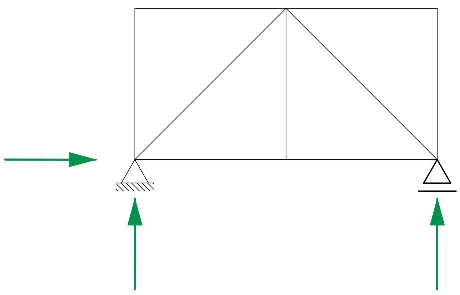
S + A = 2K = statically determinate
S + A < 2K = statically indeterminate (unstable)
S + A > 2K = statically indeterminate (overly determinate)

To have an internally statically determined truss and thus ensure its stability and stiffness, the truss must always be triangulated. In quadrangular fields one bar is missing, which leads to an unstable system. There can also be too many bars. These so-called overly determined trusses can no longer be analysed by simple means such as graphic statics.



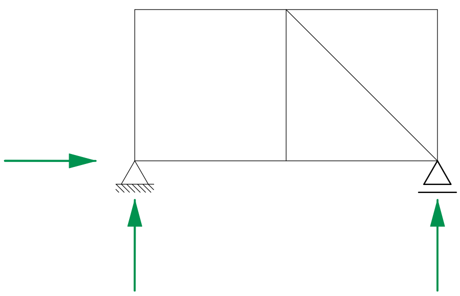
7 + 3 = 2 · 5 ✓

statically determined
= 1 possible solution for force flow



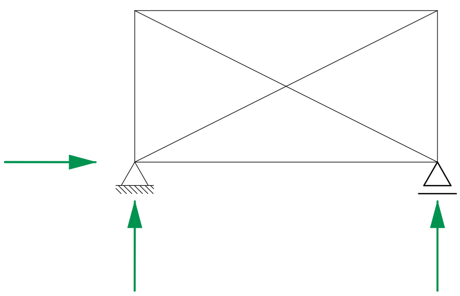
9 + 3 = 2 · 6 ✓

statically determined
= 1 possible solution for force flow



8 + 3 < 2 · 6 ×

not enough bars/members
= unstable, will collapse (not triangulated)

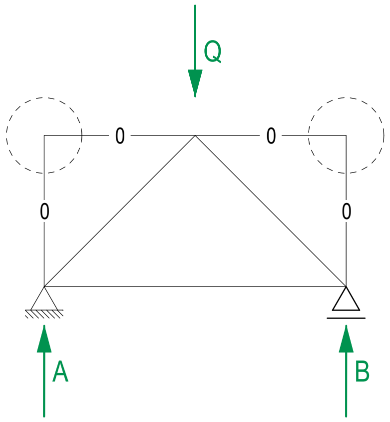


8 + 3 > 2 · 5 ×

too many bars/members
= many possible force flows

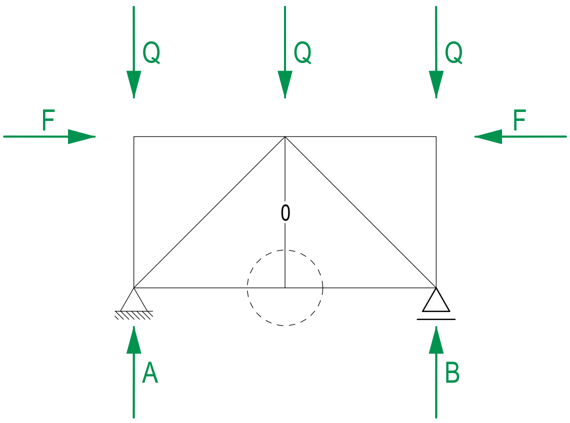
Zero Members

In trusses, certain elements don't take forces under a specific loading case. These elements are called zero members and are marked with "0" in the form diagram. But in general, due to stability reasons they cannot be removed, because as soon as the applied load changes, they will have to take up a force again. Zero members can be distinguished when constructing the force diagram where the local equilibrium in the node can be established even without the said member. The force in the member is zero, which is why there is no line in the force diagram that corresponds to the element in the form diagram. In certain situations, zero members can be identified even before the construction of the force diagram. There are three different rules for this:



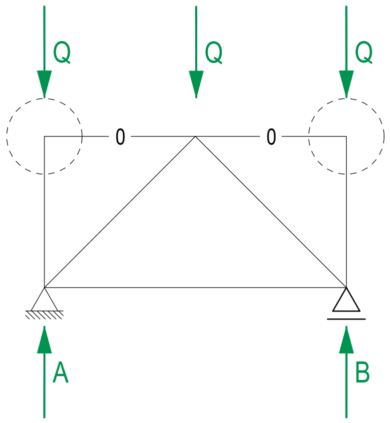
Node without force – two bars

If two bars with different directions meet, both are zero members.



Node without force – three bars

If two of the three bars are in the same direction, the third is a zero member.

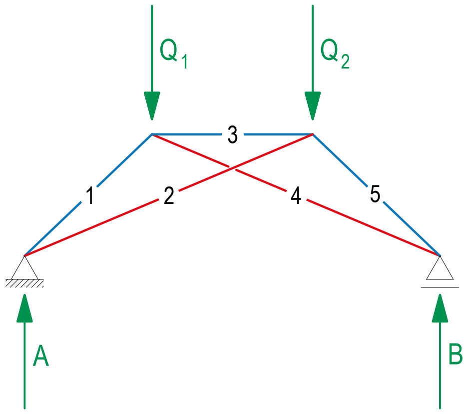


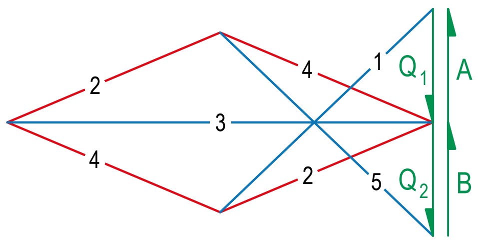
Node with force – two bars

If the load attacks in the direction of one bar, the other is a zero member.

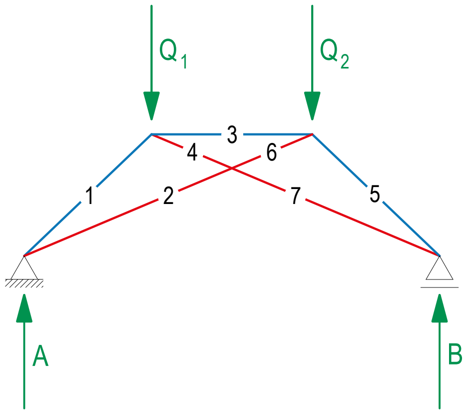
7.1 Intersecting Elements

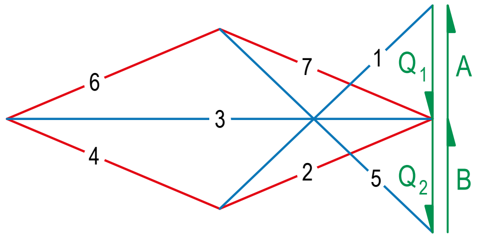
In general, the following applies in graphic statics: An element in the form diagram has one corresponding line in the force diagram. However, this is only the case as long as the elements in the form diagram do not intersect or overlap. An intersection is a fictitious node in which the elements cross in two-dimensional space but pass each other in the third dimension. Intersecting members must be drawn twice in the force diagram to create closed force polygons.



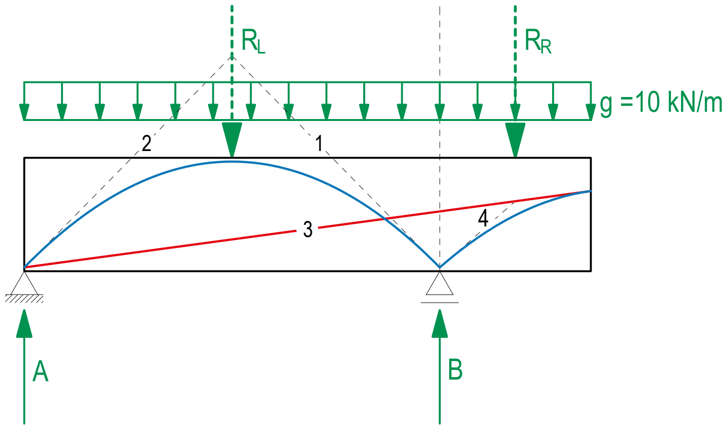


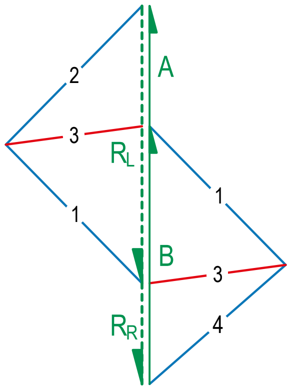
In the example above, the intersecting elements 2 and 4 form a parallelogram, i.e. a polygon with four sides in the force diagram. This intersection can alternatively be considered as a node with four elements in two-dimensional space. The node in the example below then consists of the elements 2, 4, 6 and 7, whereby 2 and 6, as well as 4 and 7, each take the same force, especially as they are each the same element in three-dimensional space. Meanwhile, nothing changes in the force diagram except the labelling.





Similar to the example above, intersections also occur with uniformly distributed loads. In the following example, the right tangent of the parabola (1) overlaps with the tension element (3). Accordingly, both elements appear twice in the force diagram.
In addition, the tension element and the right tangent of the half parabola overlap at the extreme right end of the beam. Both elements have the same force in opposite directions and lie on top of each other in the form diagram, which is why we speak of an overlap.



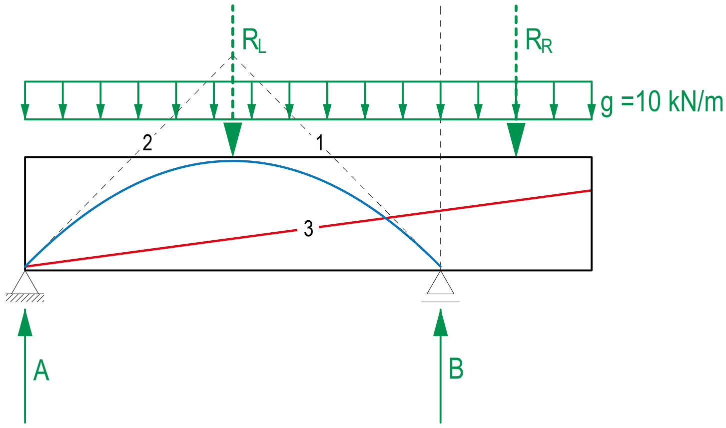


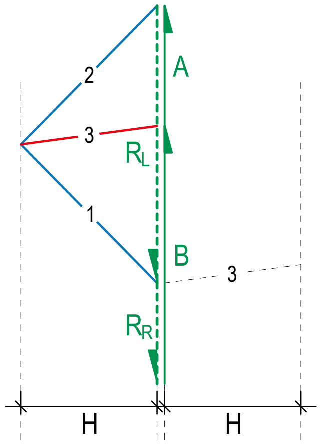
form diagrams 1:100

force diagrams 1cm ≙ 15kN

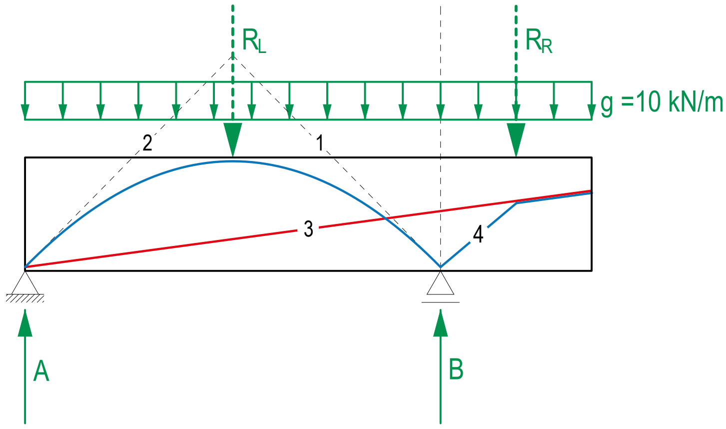
7.2 Force Flow in a Beam: Node by Node

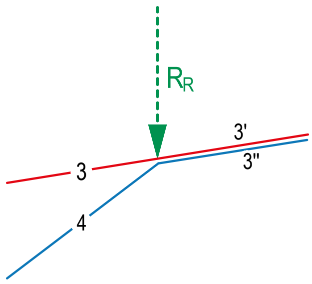
We are looking for a possible force flow in the beam that distributes the applied line load into the two supports. The given beam spans between the two supports on the one hand and also cantilevers on the right side. To solve such a combined system, the line load is divided into two subsystems. It's important to always start with the subsystem with the larger span. There, the entire structural depth is to be used. Under the given load, a parabola with the tangents 1 and 2 results in the left subsystem. With the help of the reaction force A, the inclination of the tension element (3) is then found.
In the subsystem on the right, the second parabola's horizontal thrust (H) must correspond to that of tangent 1 in order to bring the node at the support B into equilibrium. In addition, the compression arch on the right does not run into a support and must therefore be pulled back by the tension element in the same direction.



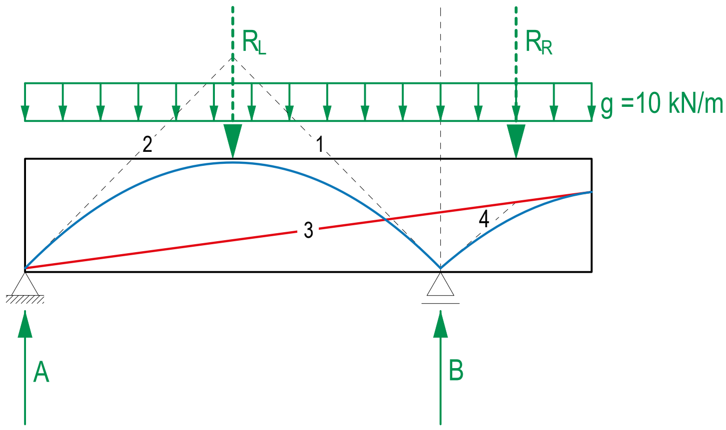


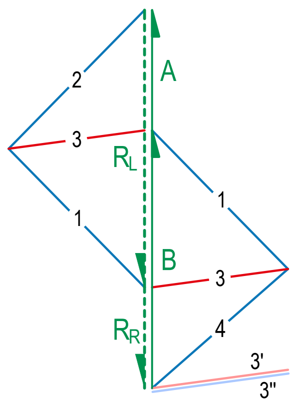
The intersection of the tension element (3) with R_R is a node with a total of five elements, whereby 3' and 3'', which cannot run into a support on the far right, must cancel each other out. Accordingly, these two elements in the node can be neglected and first the tangent (4) and then the tension band (3) are transferred clockwise into the force diagram.





Finally, all elements are brought together in the force diagram and the parabolas are drawn into the form diagram with the help of the respective tangents. For completeness, the two elements 3' and 3'' were also drawn into the force diagram in this example.



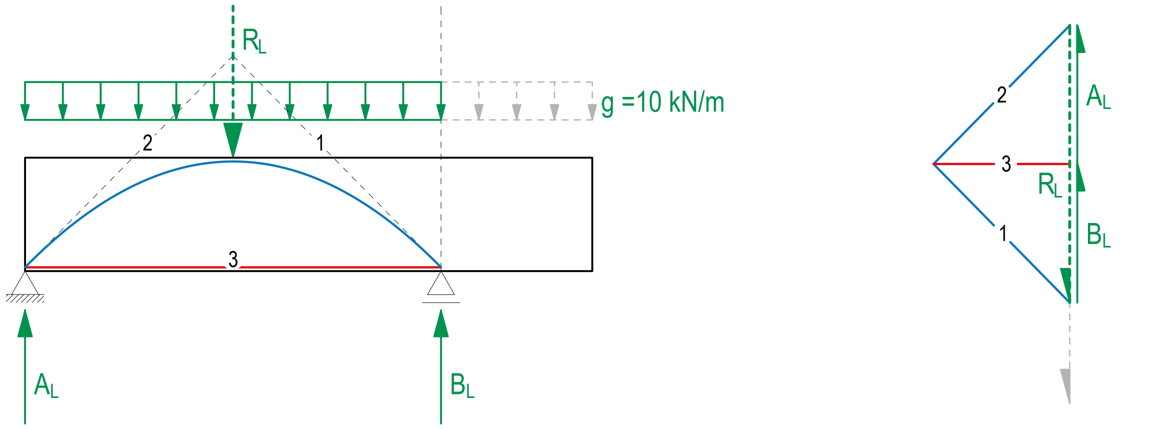


form diagrams 1:100

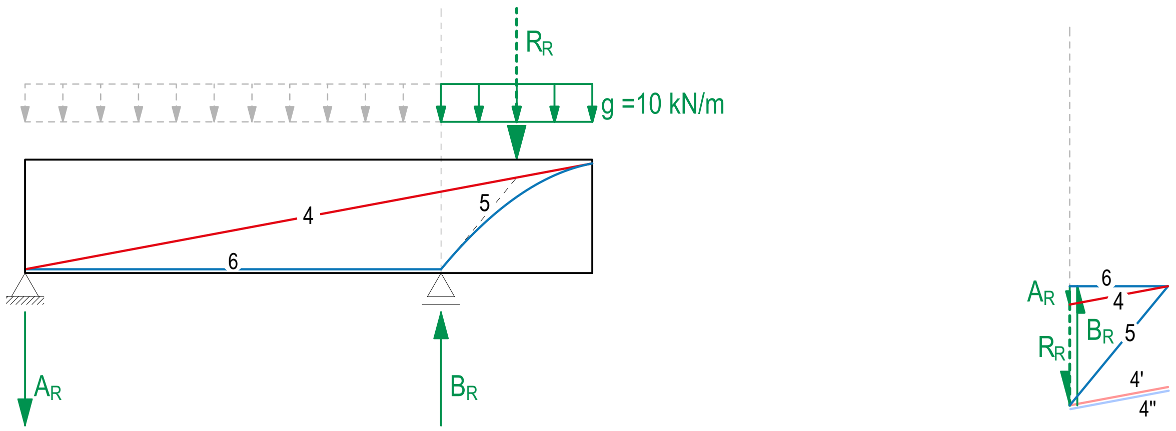
force diagrams 1cm ≙ 15kN

7.3 Force Flow in a Beam: Superposition

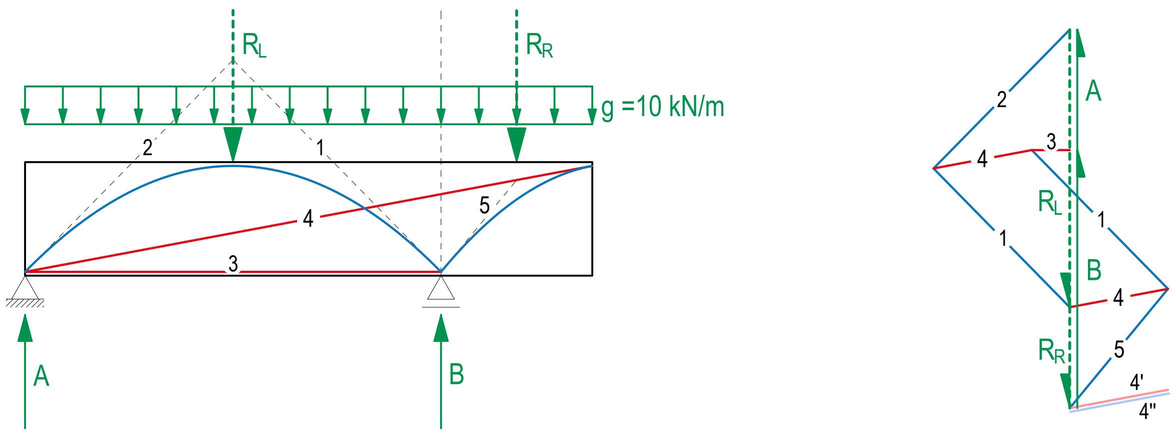
We are looking for a possible force flow in the beam that distributes the applied line load into the two supports. To solve such a combined system that both spans and cantilevers, the line load is divided into two subsystems. Both subsystems are subsequently considered separately and superimposed at the end.
First, the global equilibrium in the left subsystem is determined. Under the given load, a parabola results between the supports, whose horizontal thrust must be absorbed by means of a tension element.



Then the global equilibrium of the subsystem on the right is determined. The parabola that occurs under this load does not run into a support on the far right side of the beam. This compression force must therefore be pulled back by a tension element. It is important that the direction of the right tangent of the arch corresponds to that of the tension element and that the two elements thus cancel each other out. To compensate for the horizontal component of the arch in support B, an additional compression element is needed between the supports.



Finally, the two subsystems are graphically superimposed. In the force diagram, the global equilibrium of the two subsystems can first be added and then the internal forces can be combined node by node. From the two force diagrams above, it can be seen that the element between the supports absorbs a larger tension force, which is why the superposition must also be a tension element (3).

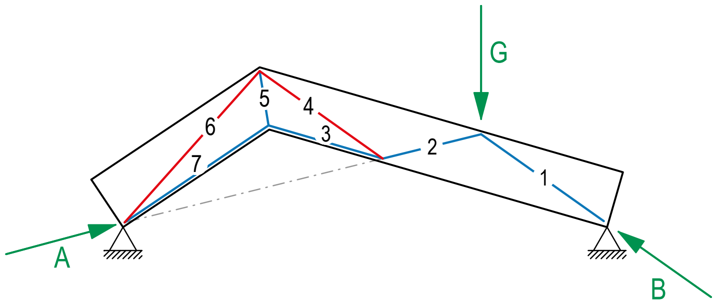


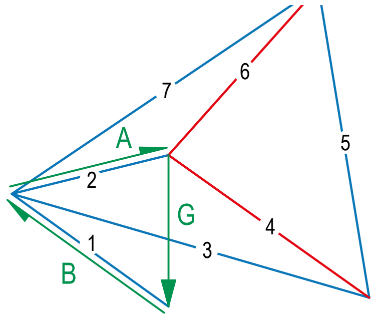
form diagrams 1:100 *force diagrams 1cm ≙ 15kN*

In order to develop possible, necessary deflections of the internal forces in beams, frames or wall panels, it is useful to start from the direct course of the applied force into the supports. With this so-called thrust line, the global equilibrium is found. However, since the thrust line may run outside the material, the internal forces must be redirected in such a way that they run through the material.

Frame corner

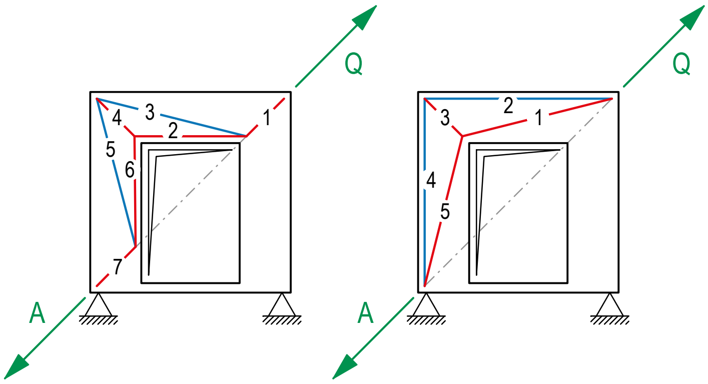
First, the global equilibrium is found with the help of the thrust line. Then, looking at the internal force flow, element 1 follows the thrust line and directs the applied force on the right-hand side directly into support B. On the left-hand side, however, the thrust line runs outside the material, which is why this compression force must be redirected there. Therefore, from where it would leave the material, it is guided into support A with the help of two tension cables and an additional compression element.

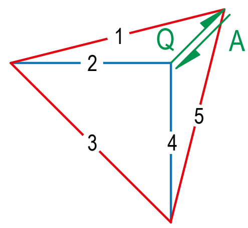




Wall with opening

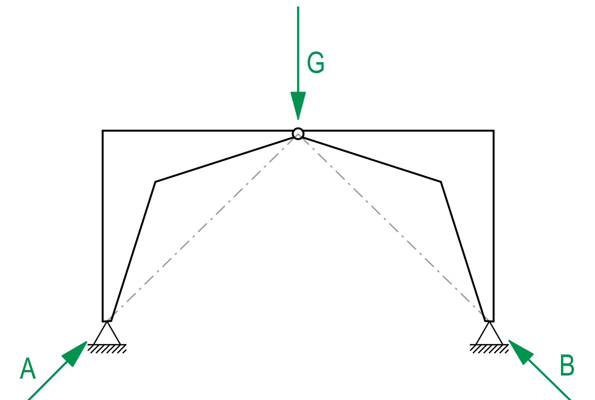
In the lower left example, the force flow follows the thrust line, equal to the frame corner above, as long as it remains inside the material and is deflected from where it leaves the material. For comparison, the example on the right shows the simplest type of deflection, consisting of two cantilevers and a frame corner. In this way, elements can be spared in the form diagram and thus in the force diagram. This simplest deflection can be used likewise for both, compression and tension elements.



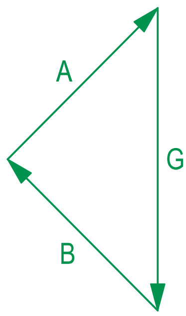


Hinges and global Equilibrium

The lines of action of the applied force and those of the reaction forces (thrust line) always intersect at the hinge in the form diagram, since the sum of all forces equals zero there. This helps us to find the global equilibrium.



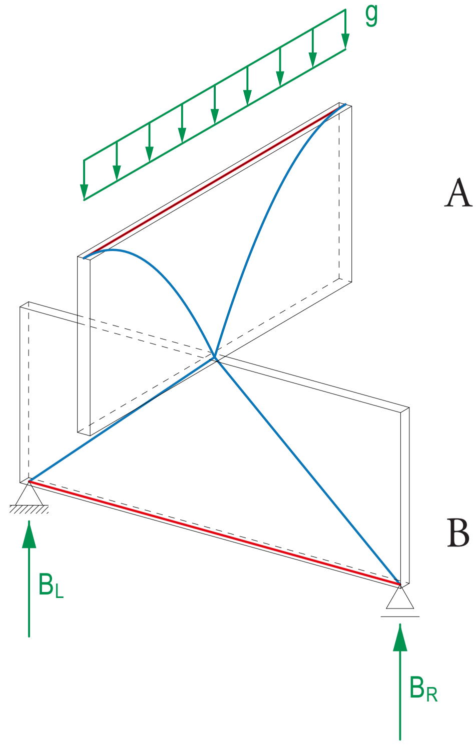
form diagrams 1:100

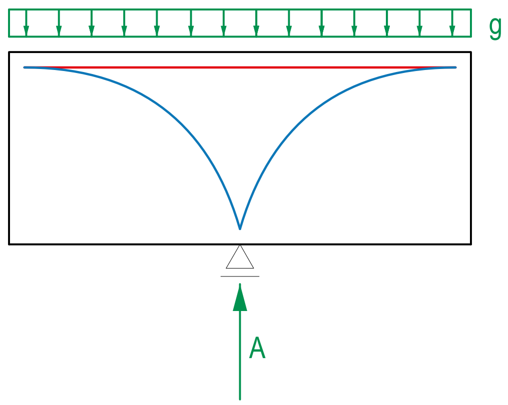


force diagrams 1cm ≙ 10kN

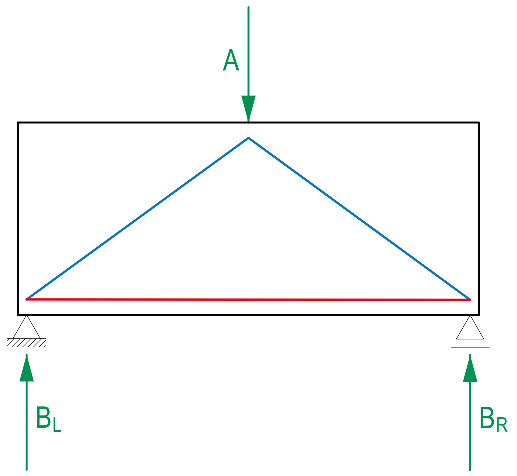
A possible force flow that transfers the applied line load through the upper slab onto the lower one and from there into the two supports is to be found.

To solve such a system, the two slabs are considered as individual subsystems: The upper slab with the applied line load is the first subsystem (A), and the lower slab with the two supports is a second subsystem (B). Their point of contact is understood as the support of subsystem A and as the point of application of the reaction force of the upper subsystem on the lower subsystem.





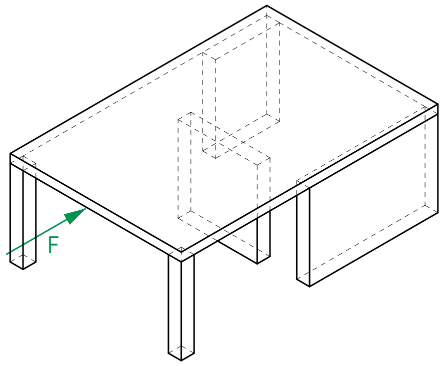
Both subsystems are solved separately. A line load acts on the upper slab (subsystem A) and there is one support. For the given situation, the force flow is to be drawn as shown. The internal forces and the reaction force A are determined in the corresponding force diagram. Finally, the force flow is sketched qualitatively in the axonometric drawing.



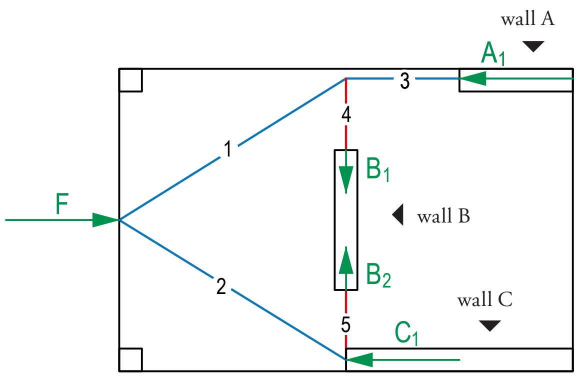
If the lower slab (subsystem B) is now considered as a separate system, subsystem A acts as an external force on their point of contact. The magnitude of this force corresponds to the reaction force of subsystem A. When changing the system, however, the direction of the force must be reversed. The applied force is finally transferred to both supports via two compression elements. With the magnitude of the force A already found, the force diagram can be completed and the force flow can again be qualitatively transferred to the axonometric drawing.

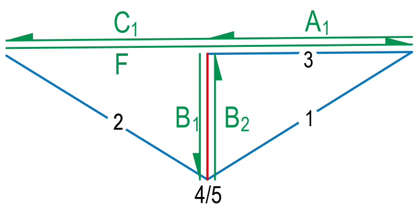
10.1 Compendium Structural Design I&II
Horizontal Forces

A possible force flow in a slab is sought, which brings the horizontally acting load into the vertical load-bearing elements. To solve such a situation, the elements (slab and walls) are considered as individual subsystems.

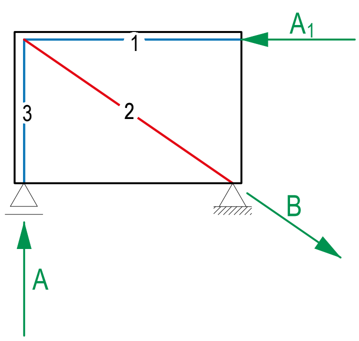


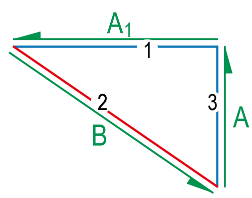
Since walls can only absorb forces along their axis, the possible lines of action of the walls and thus their points of intersection are drawn into the slab first. If more than one intersection is found, the system is properly braces. If possible, the applied force is guided into the intersection points found and can then be redirected into the axes of the walls. In the example below, the force F is guided into the intersection points via two pressure elements. From there, the compression force is redirected in the direction of wall B with one tension member each and introduced into walls A and C.
Finally, the external forces acting from the walls on the subsystem «slab» are added to the internal force flow and their magnitude is determined in the force diagram. The force flow in the ceiling is in equilibrium when all external forces in the force diagram cancel each other out.





In order to now bring the horizontal forces in the slab via the walls into the floor, a corresponding force flow must be found in each wall.
Wall A is viewed from above (see arrow), which means that the force applied by the ceiling occurs on the right side of the wall. The magnitude of the applied force A_1 corresponds to the reaction force A_1 found in the subsystem «ceiling». Again, the direction of the force must be changed when switching between the subsystems.
So now a force A_1 pushes on the wall from the right. This force is then led all the way to the left through the wall, where it is redirected by a tension element and finally runs vertically into the roller support A. The tension element can be guided directly into the second support B. The effect of the force A_1 finally results in the reaction forces A and B. The same can be done with the other two subsystems of walls B and C.





form diagrams 1:100

force diagrams 1cm ≙ 10kN

10.2 Compendium Structural Design I&II
Buckling

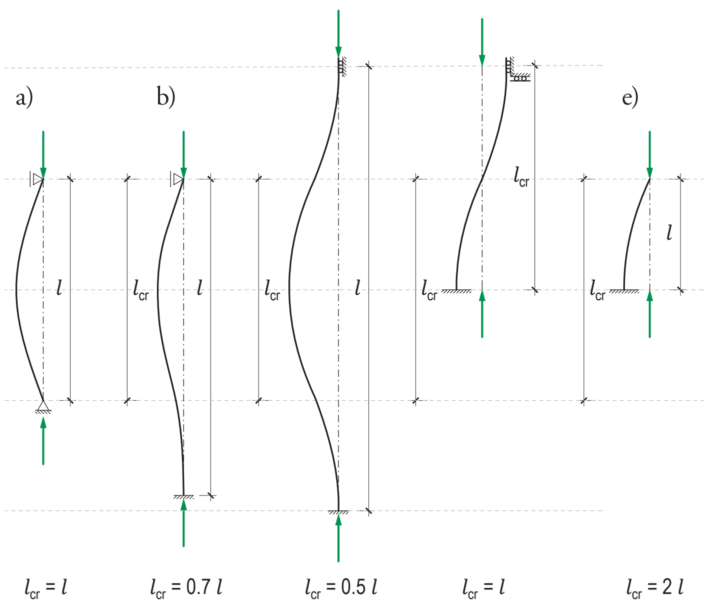
When dimensioning columns, the necessary cross-sectional area is calculated by means of the relevant (maximum) force acting on the element. However, if an element is to be very long and thin, i.e. very slender, there is a risk that it will buckle under the compressive force. In this case, before material failure can occur, buckling failure occurs, which in this respect is a problem of geometric nature. To counteract this, a more suitable cross-section could for example be chosen for the column.

In the diagram below, the x-axis shows the critical length l_{cr} over the square root of the cross-sectional area A.

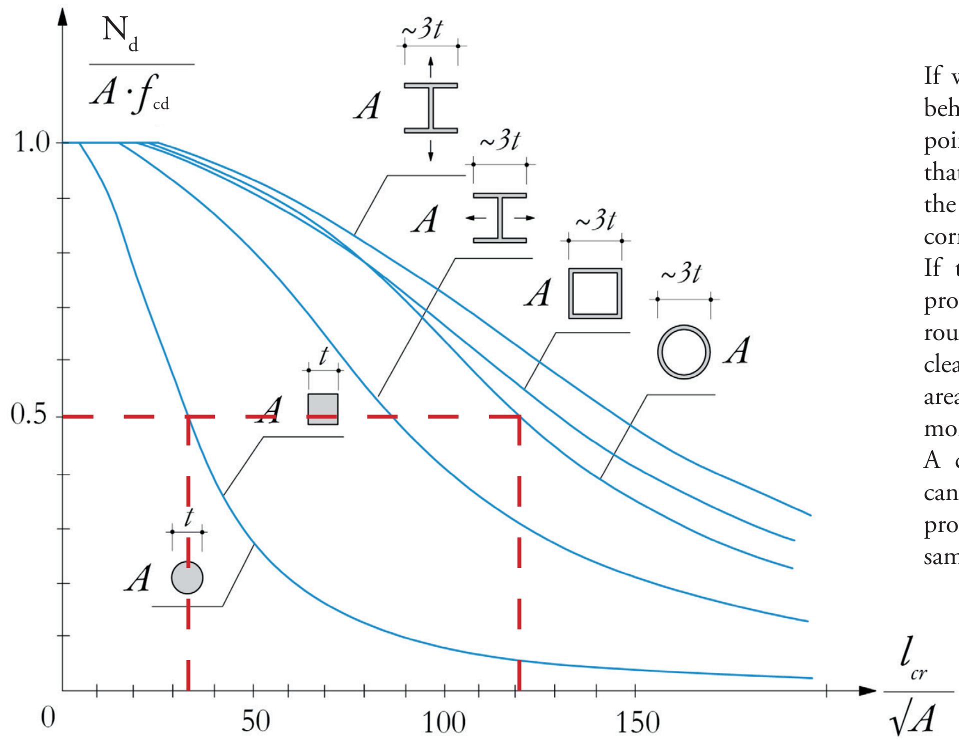
The critical length depends on the support of the element. A clamped support leads to a shorter critical length than a hinged support. The diagram on the right shows the following boundary conditions:

a) hinged support on top and bottom
b) fixed at the bottom, hinged on top
c) fixed on top and bottom
d) fixed at the bottom, fixed roller on top
e) fixed at the bottom, no support on top

The y-axis shows the relevant force N_d over the cross-sectional area A multiplied by the compressive strength of the material f_{cd}. The higher this value, the more likely a component is to buckle.



Critical length according to the support conditions



Buckling curves of steel profiles

If we look at the curves of the buckling behaviour of different cross-sections, every point that lies below the curve means that the profile holds; every point above the curve equals a buckling failure of the corresponding profile.
If the buckling curve of a round solid profile is now compared with that of a round hollow profile, it quickly becomes clear that for the same force, cross-sectional area and material the solid profile is much more likely to buckle than a hollow profile. A column designed as a hollow profile can be almost four times as long as a solid profile column before it buckles under the same load.