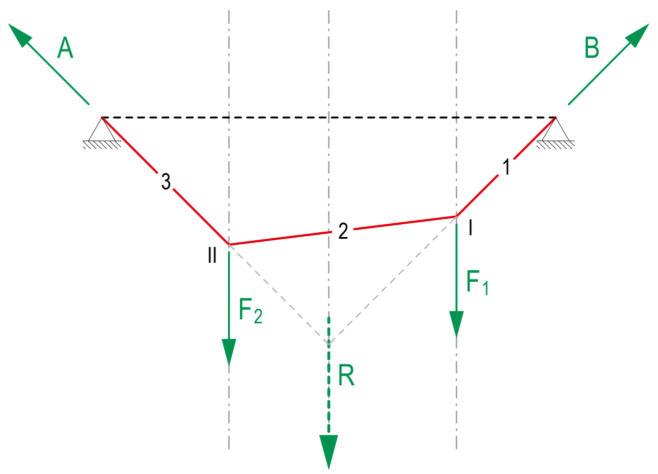
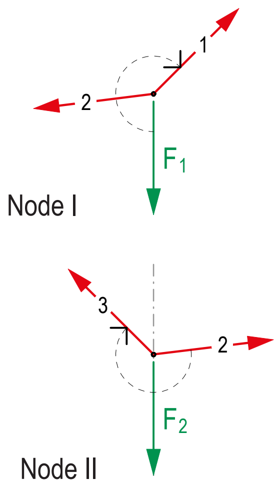
Structural Design I

Form Diagram - Force Diagram - Free Body Diagram

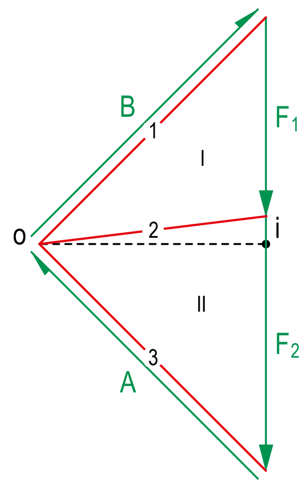
In graphic statics, forces of a structure are shown as vectors, and therefore two plans (form and force diagram) are being used. The form diagram shows the geometry of the structure to scale with all bearing elements and the location of the loads. The force diagram shows the external and internal forces to scale. Every line in force diagram corresponds to the parallel line in form diagram. The free body diagrams are sketches not to scale and show information about single nodes.



Form Diagram - scale 1:200



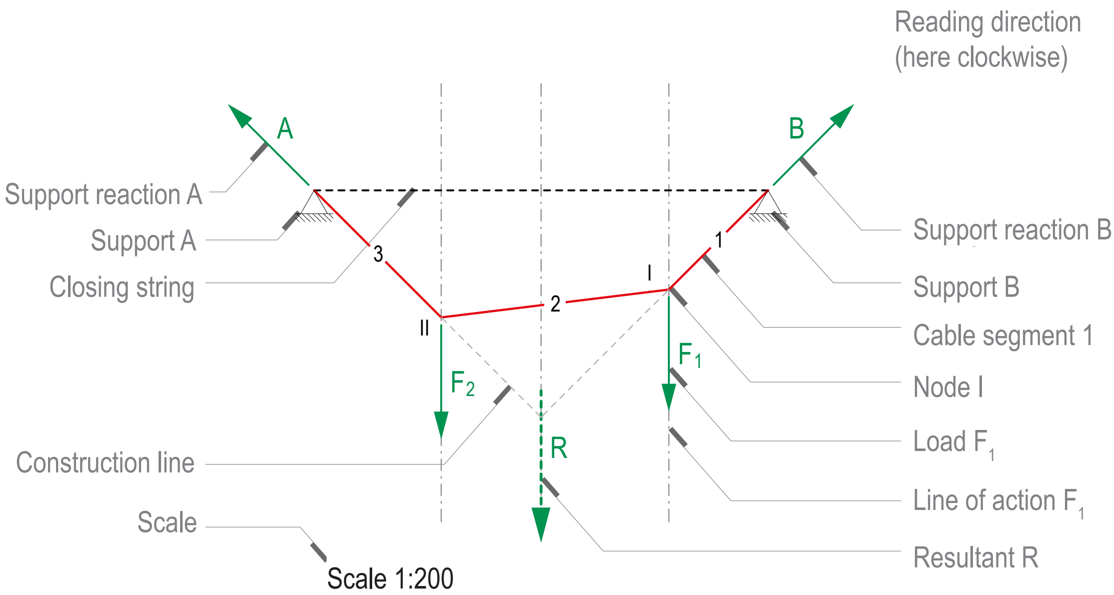
Free Body Diagram - no scale



Force Diagram - scale 1cm ≙ 50kN

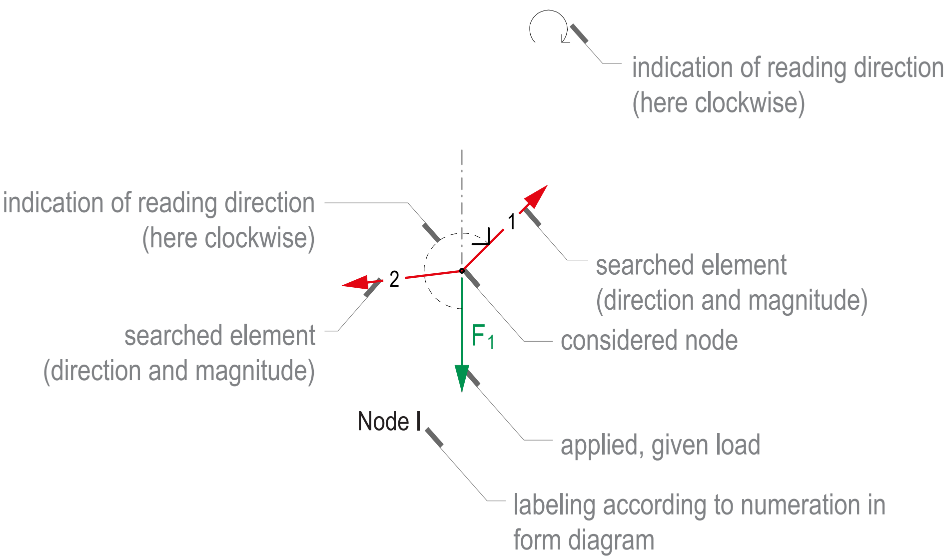
Form Diagram

The form diagram shows the geometry of the structure with all bearing elements and the location of loads. Loads (F_1 and F_2) and support reactions (A and B) are so called “external forces”and are shown with their direction (as an arrowhead). They are drawn in green colour. Forces within the structural elements (cable elements 1-3) are called „internal forces“ and have no distinct direction. Depending on the kind of load they are shown in red colour for tension or blue for compression. The form diagram has to be drawn to scale, e.g. 1:200 means that 1cm in plan equals 200cm in reality.



Free Body Diagram

The free body diagram (FDB) and the indicated reading direction declare the order of the elements that are to be drawn in the force diagram. The FDB is a sketch and thereby it is not to scale. It shows all forces and elements applied on the respective node and differentiates between given and searched elements. By transferring the direction of a force from force diagram to FDB, one can indicate tension (direction away from the node, usually colored red) and compression (direction towards the node, usually colored blue) elements.

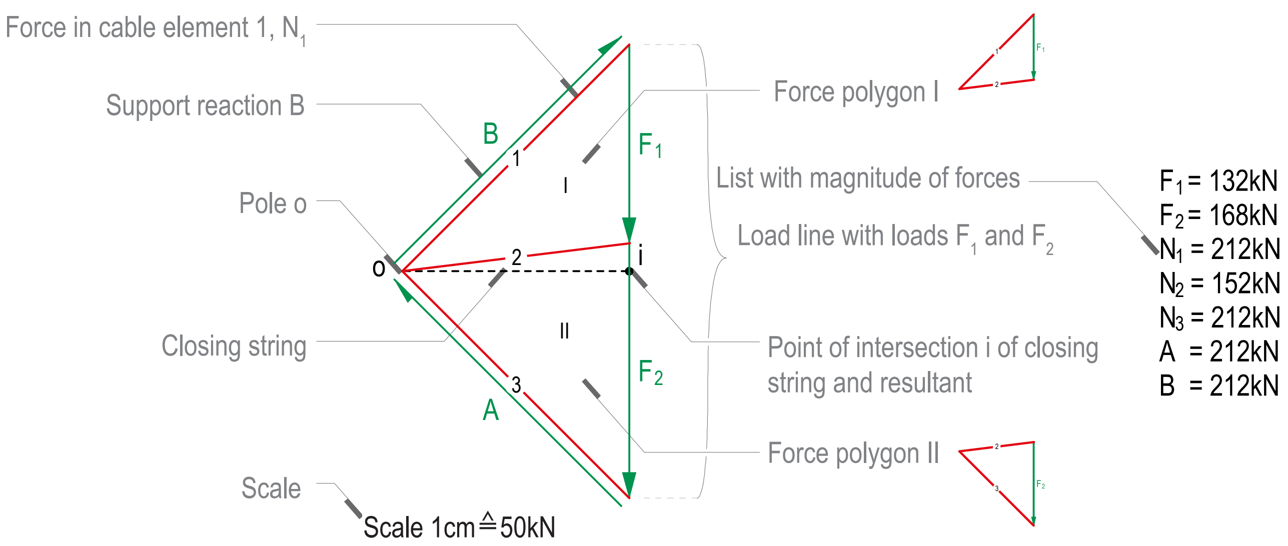


Force Diagram

The force diagram is constructed by parallel moving the elements from the form diagram. The chosen reading direction and the respective free body diagram indicate the drawing order of the elements. Once the force diagram is drawn to scale, the magnitude of the forces can be measured, e.g. 1cm = 50kN means that 1cm in the force diagram equals a force of 50kN.

Global and Local Equilibrium

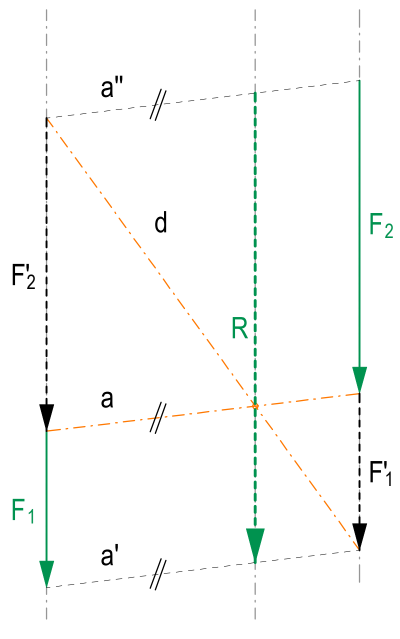
The load line is a force polygon of all external forces (action and reaction forces). The total system is in equilibrium (global equilibrium), if the polygon is closed. One separate node (free body diagram) is in equilibrium state if all forces, that affect this node, form a closed polygon in force diagram. The forces polygon is defined as a model for local equilibrium of the internal forces.



F2 Structural Design I
Resultant of Parallel Forces

We are looking for the position of the resultant of two forces with parallel lines of action. It is situated in the centroid of two forces. The position of the resultant is closer to the bigger force, following the proportion F /F = d /d.

As a first step, the forces should be drawn in form diagram with a chosen scale (e.g. 1cm corresponds to 1 kN). In the drawing below F_1 and F_2 are drawn in green. Then we need to connect the starting point of the first force with the endpoint of the second force (line a in example below). This line now should be shifted to the end point of the first force (a‘) and to the starting point of the second force (a“). The result is a parallelogram. In this parallelogram we draw the diagonal d, whose action line begins on the action line of the first force and ends on the action line of the second force. In the intersection point with the line a the resultant R is situated.



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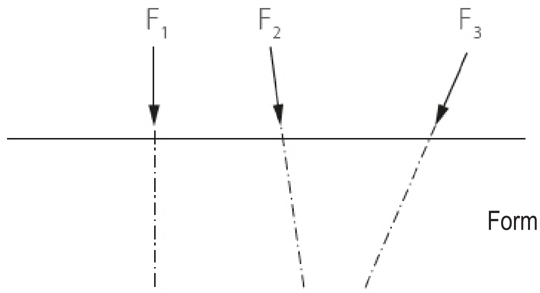
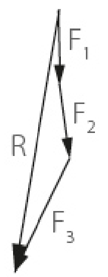
Structural Design I

The Resultant of a Set of Non-Concurrent Forces

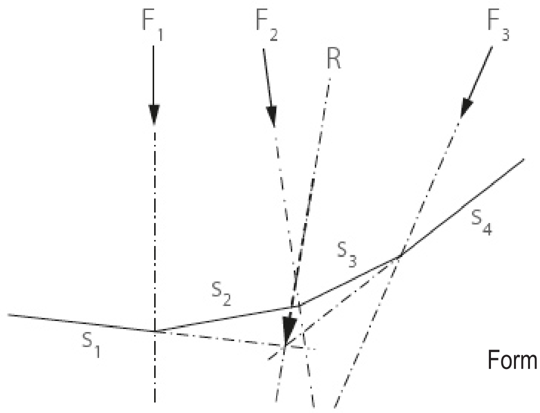
Three forces of different directions are given (form diagram a), the magnitude and the position of the resultant of these three forces should be found. First we need to construct a load line in force diagram (force diagram a).The connection between the starting point and end point of the load line gives the magnitude and the direction of the resultant R. Then a trial polygon (funicular polygon) is drawn in order to define the position of the resultant. The trial funicular could be represented as a suspended cable (form diagram b) or as an arch (form diagram c). Pole o is chosen freely. The rays S_1 - S_4 constructed in this way are moved parallely to the form diagram. The extensions of the first and the last segments intersect in the point in form diagram, through which resultant has to pass as well. This is one possibility from infinite number of solutions. Nevertheless, the position of the resultant is always the same.



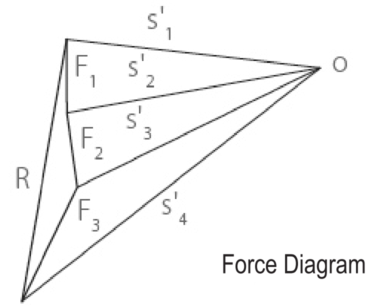
Form Diagram a)



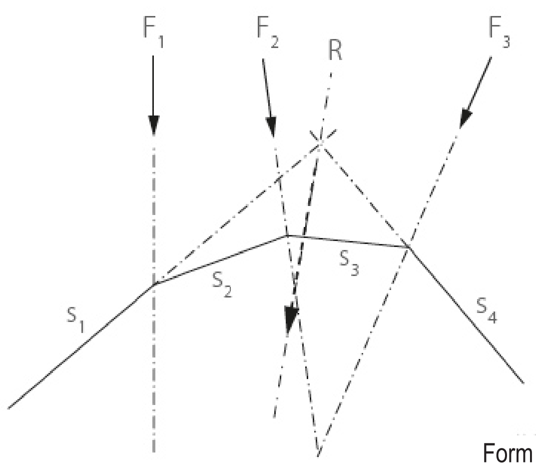
Force Diagram a)



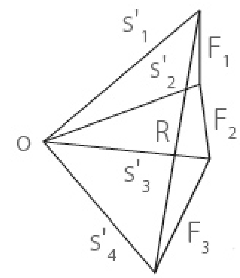
Form Diagram b)



Force Diagram b)



Form Diagram c)

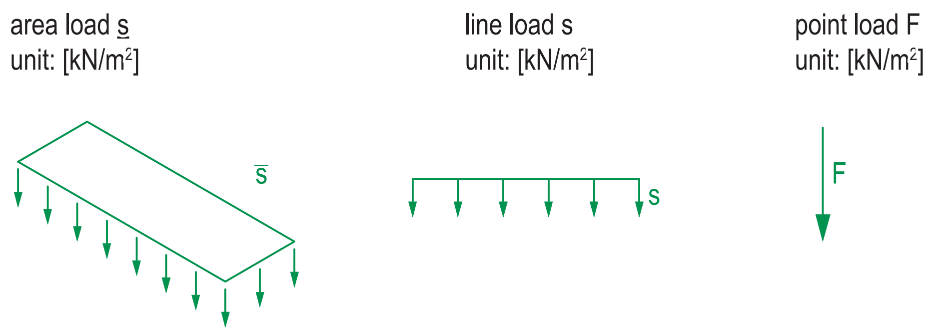


Force Diagram c)

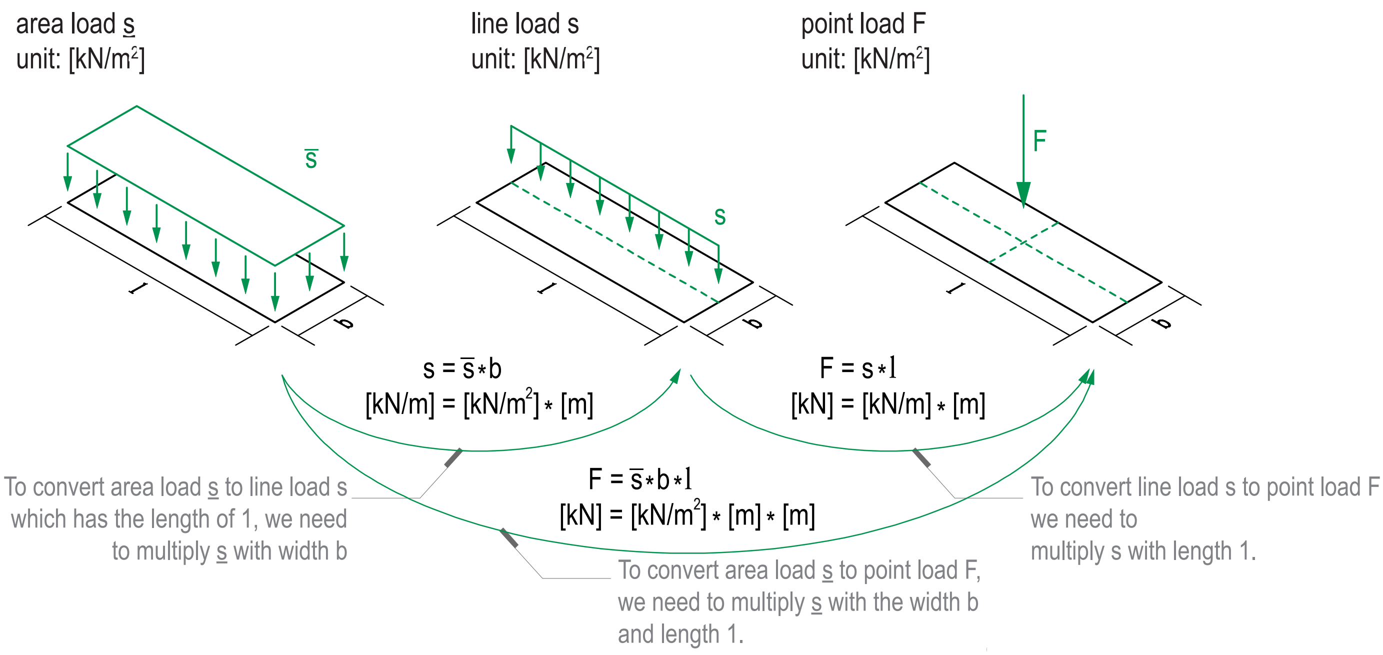
Types of Loads

We differentiate between three different types of loads: point load (e.g. columns), line load (e.g. walls) and area load (e.g. snow).

To know what kind of force is given, we use different indications: point loads in capital letters, line loads in lower case and area loads in coated lower case.



Conversion of Different Types of Loads



Safety Factors

The loads are divided into self weight (dead / constant load) and live load (changing load). The self-weight (roof structure) can be accurately defined, the live load (snow, wind etc.) are complicated to determine. This is the reason for different safety factors to be used:

Safety Factor Dead Load	$\gamma_G = 1.35$
Safety Factor Live Load	$\gamma_Q = 1.50$

We differentiate between characteristic force value (index K), which has not been multiplied with safety factor and design force value (index d), which is calculated taking safety factors into account:

	Point Load	Line Load	Area Load
Dead Load	$G_d = G_k \bullet \gamma_G$	$g_d = gk \bullet \gamma_G$	$\overline{g_d} = \overline{gk} \bullet \overline{\gamma_G}$
Live Load	$Q_d = Q_k \bullet \gamma_Q$	$q_d = qk \bullet \gamma_Q$	$\overline{q_d} = \overline{qk} \bullet \overline{\gamma_Q}$

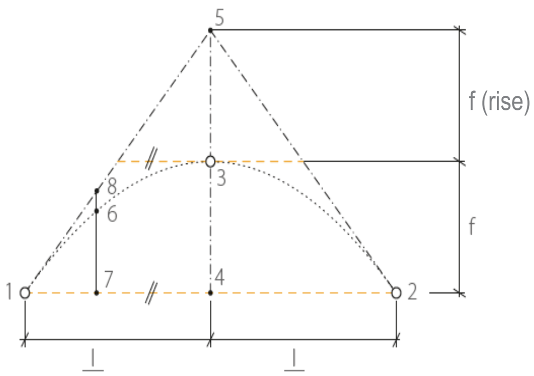
Construction of a Parabola - under Distributed Load

Construction of a Parabola

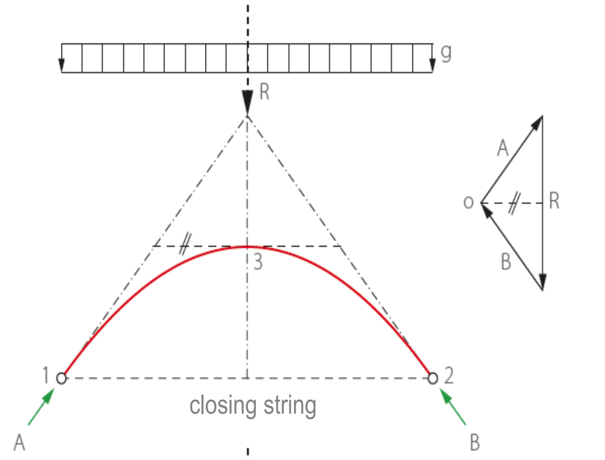
In case when the parabola goes through points 1 to 3, and the distances 1-4 and 4-2 are equal, tangents 1-5 and 2-5 of the parabola cross with vertical line through point 3 in a way that distance 5-3 is equal to 3-4, that again corresponds to rise f. The parabola tangent at point 3 is parallel to closing string 1-2. In general the following equation is true: 8-6 to 6-7 is equal to 1-7 to 7-2.

Crown Point of the Parabola

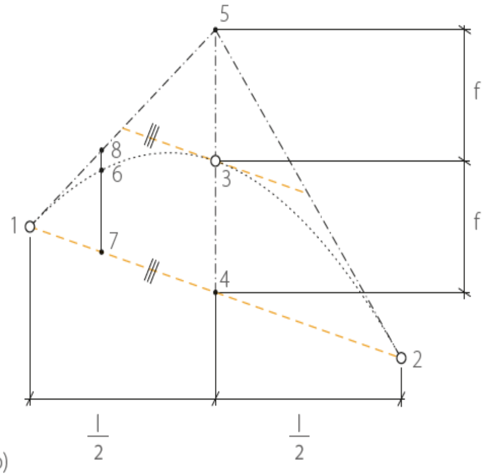
To construct a crown point of the parabola, the support reaction A should be mirrored horizontally at the pole o in force diagram (d). Then the following is true: R1 / R = l1 / l , the action line A‘ can be shifted to the form diagram. In this way defined triangle A-A‘-R1 corresponds to symmetric parabola, that is situated in geometrical center.



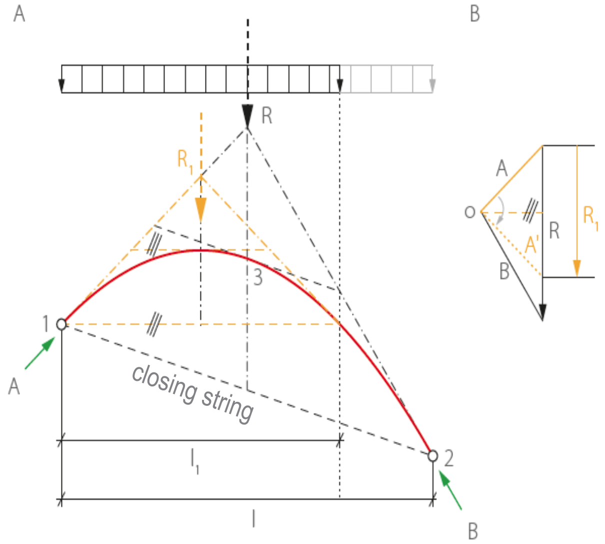
(a)



(c)



(b)



(d)

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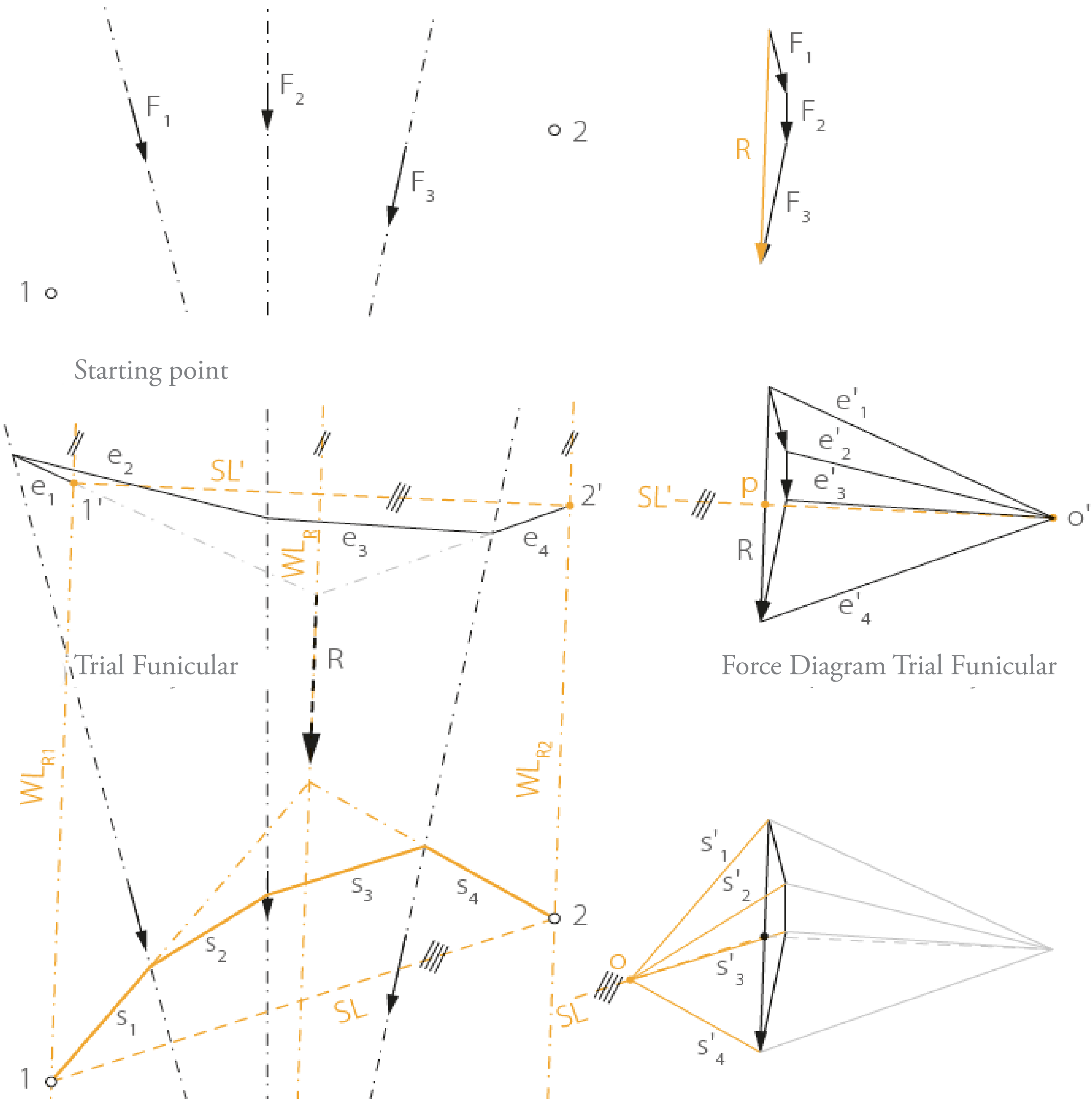
Structural Design I

Finding the Thrust Line by Means of a Trial Funicular

The example shows how to determine the thrust line (cable form) between two points, which takes the forces F1 to F3, with the help of a trial funicular. In the first step, the direction of R is to be determined. The line of action of the resultant, WL_R, is shifted parallel through the two supports, creating WL_{R1} and WL_{R2}. Then, a trial funicular starting with WL_{R1} is constructed. The trial closing string SL‘ (between points 1‘ and 2‘) is shifted parallel through the pole of the trial force diagram.

The intersection point p of SL‘ with R in the force diagram is constant, regardless of the shape of thrust line. Thus, the closing string SL can be shifted parallel through this point in force diagram. Any desired point on this line can be chosen as pole o and corresponding rays s‘1 to s‘4 can be transferred to the form diagram. Thus, the thrust line through points 1 and 2 can be obtained.



Starting point

Trial Funicular

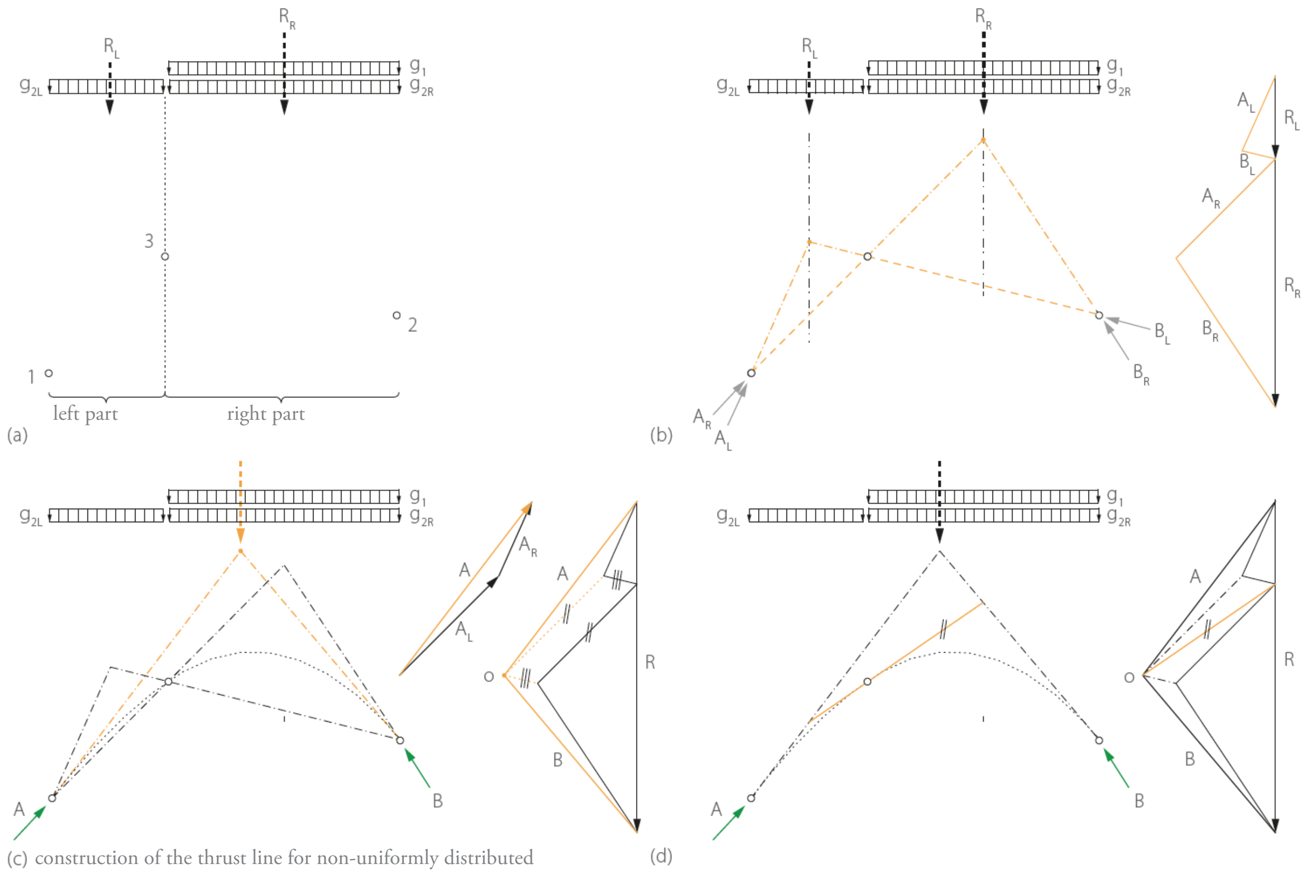
Force Diagram Trial Funicular

Form Diagram

Force Diagram Thrust Line

Thrust Line for Non-Uniformly Distributed Loads using Superposition

The goal is to find the thrust line that passes through points 1, 2 and 3 (a) for the given load (a). In the first step, the system is divided into two parts: the section to the left of point 3 with the line load g_{2L} and the section to the right of point 3 with the line loads g_1 and g_{2R}. The next step is to determine the thrust line for each side which can take the resultant of the line load acting on that side and passes through the three points (b). By drawing lines parallel to A_R and B_L in the force diagram, the new pole o, their intersection point, is obtained (d). The support reactions A and B are determined by connecting the pole o to the start- and endpoint of the load line. By means of parallel shifting A and B from the force to the form diagram, the tangents required to create the thrust line for the given load is obtained. The tangent in point 3 is determined by connecting the end point of R_L with the pole o in the force diagram and transferring this line to the form diagram.



(a) (b)

(c) construction of the thrust line for non-uniformly distributed load

(d)

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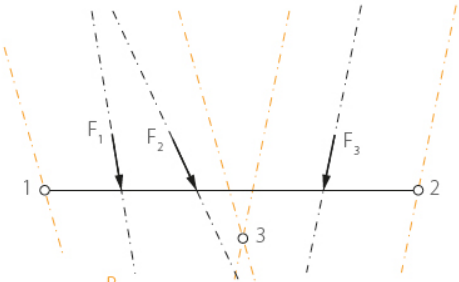
Structural Design I

Thrust Line for Any Loading Case by Partial Closing Strings

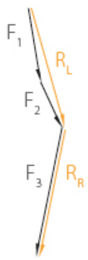
The thrust line for any set of point loads can be found using partial closing strings. In the first step, the respective resultant for the loads between points 1 and 3 and the loads between points 2 and 3 are determined: R_L and R_R.

The lines of action of the reaction forces at points 1, 2 and 3 are parallel to their respective resultants (a). In the next step, poles o'_L and o'_R are chosen for each of the two systems and the force diagram as well as the corresponding auxiliary constructions (in orange) are drawn in the form diagram. Parallel lines to the closing strings of the two funicular polygons are drawn through the pole in the respective force diagram, resulting in two intersection points with the respective resultants i_L and i_R (b).

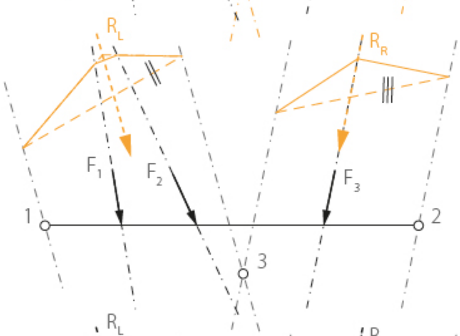
The closing strings of both systems between points 1 and 3 as well as between points 2 and 3 are drawn and parallel lines to the closing strings are drawn through points i_L and i_R in the force diagram. The intersection point of these lines is the pole o. The force diagram is completed and the funicular polygon is drawn in the form diagram, starting from point 1, and passing through points 3 and 2 (c).



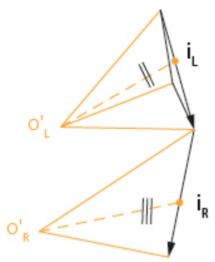
Form Diagram (a)



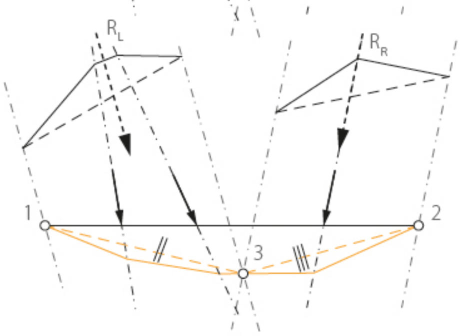
Force Diagram (a)



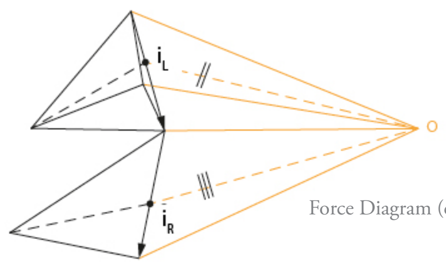
Form Diagram (b)



Force Diagram (b)



Form Diagram (c)



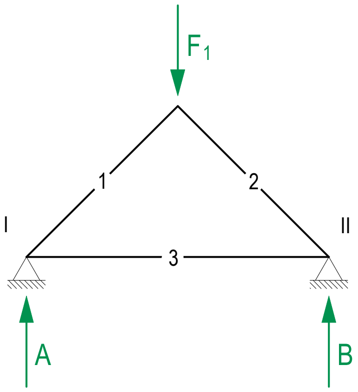
Force Diagram (c)

Determining Tension and Compression Forces

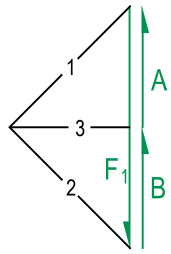
In order to determine the types of stress, the local equilibrium of the node in the predefined reading order is considered and the respective polygon is drawn. The easiest and fastest way of doing this is to draw it while constructing the force diagram.

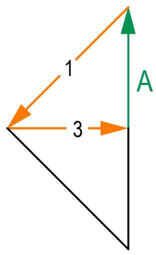
If the forces are applied to the node, then:
In case the force is acting in direction away from the node, it is a tension force.
In case the force is acting in direction towards the node, it is a compression force.

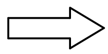
Form Diagram

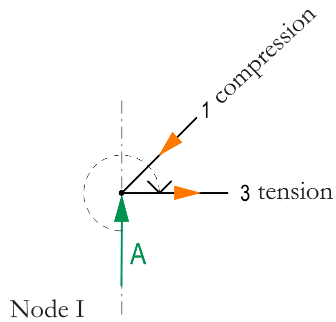


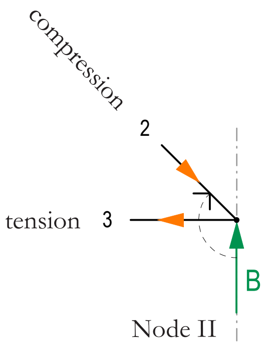
Force Diagram

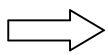


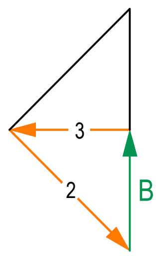


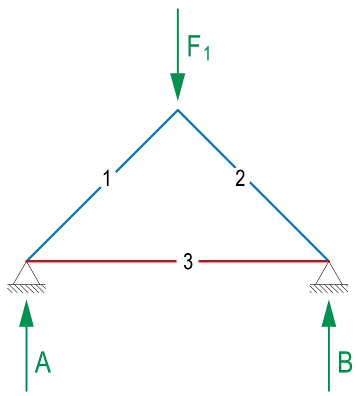












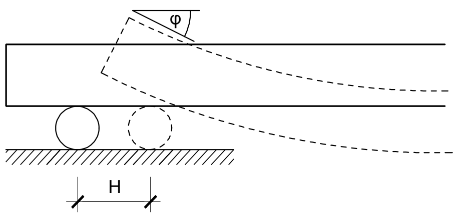
The result is always shown in form diagram. Tension elements are indicated with red and compression elements with blue colour.

Supports

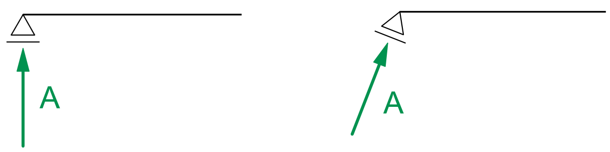
The supports are defined as the parts of structure where it is supported by other structures or by the building ground. Depending on the way of how the structure is being supported, different types of reaction forces emerge. These depend on the different number of possible movements of the structure at the support.

Roller

A roller allows both rotation (φ) and a movement H of the support. A roller can only take a force A which is perpendicular to the shifting direction of the support. In general it is directed vertically.

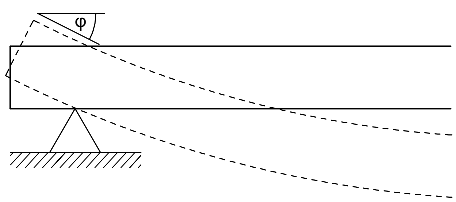


Representation in the static system

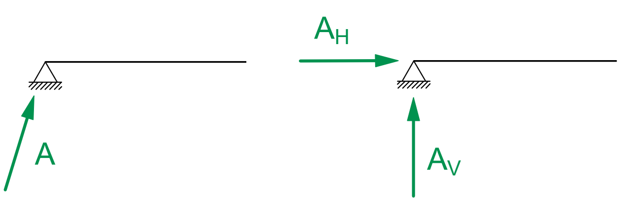


Pin or Hinge

Pins and hinges allow rotation (φ) only, but no horizontal or vertical movement of the support. The support reaction can be divided into two components, A_H and A_V.

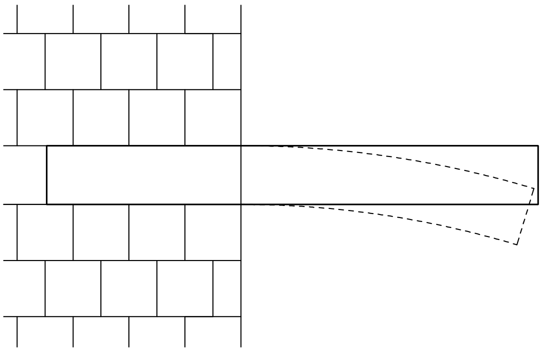


Representation in the static system

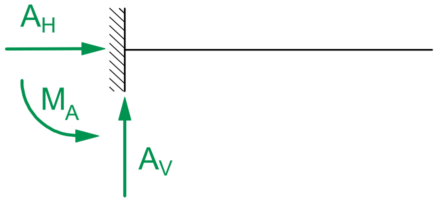


Fixed End Support

In the case of the fixed end support, neither a rotation (φ) nor a movement H of the support is possible. In this case, besides horizontal and vertical support reaction forces, also the fixed-end moment M_A emerges.

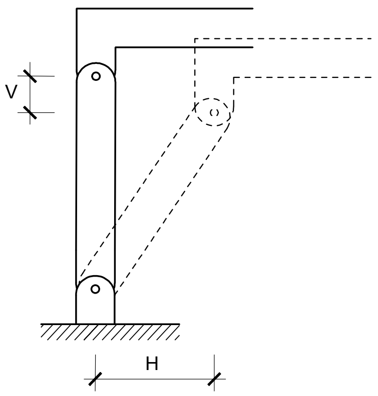


Representation in the static system

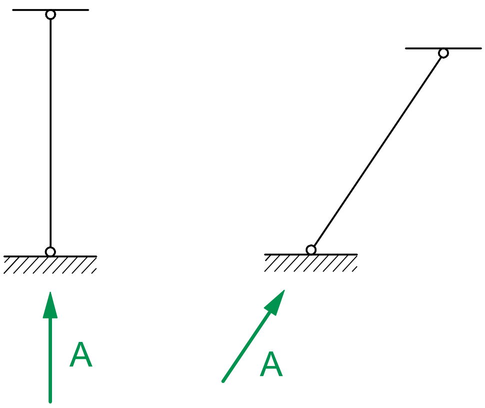


Hinged Column

A special type of support is the so-called hinged column. Its mode of action is shown on the drawing below. It corresponds to the one of the roller. In this case, only a force A that passes along the direction of the hinged column can be taken.



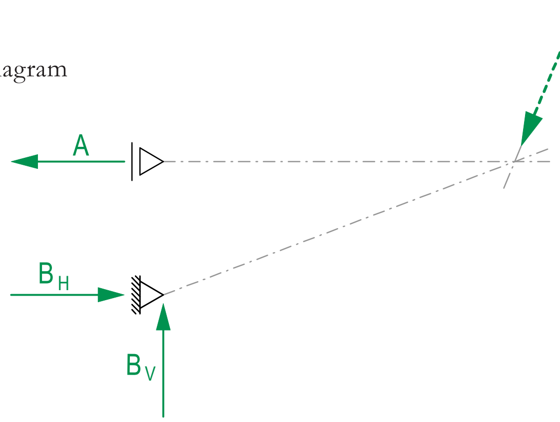
Representation in the static system



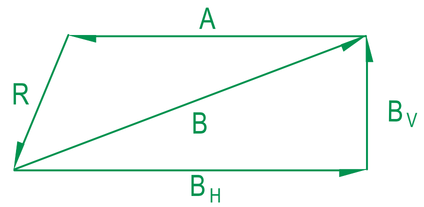
General Case

The general rule is the following: If the action lines of the resultants and the action lines of the support reaction forces intersect in one point in form diagram, the reaction forces can be determined graphically with the help of the resultant.

Form Diagram



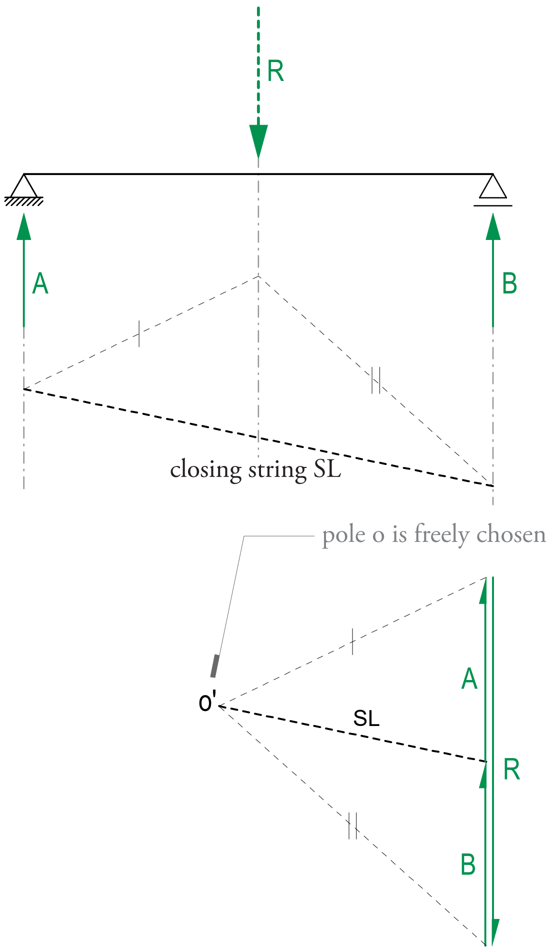
Force Diagram



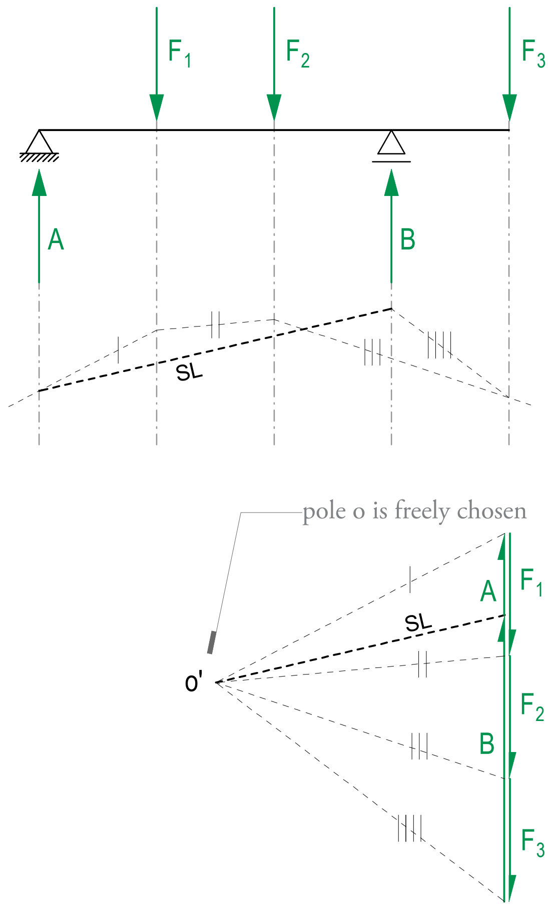
Parallel Forces

If the action line of R is parallel to the action lines of the reaction forces, there is no direct solution. In this case, it is required to work with an auxiliary system (trial polygon). In the first step, the load line is drawn in force diagram without reaction forces. Then, a pole o is chosen and segments are drawn from o to the points of the load line in the force diagram. These segments are then transferred to form diagram. As per the rule, according to which elements that form a closed polygon in the force diagram, intersect in one point in the form diagram, the trial polygon is drawn in form diagram. Thus, the found closing string is transferred to the force diagram, which allows to determine the reaction forces A and B.

Case 1: Position of the resultant is given



Case 2: Position of the resultant is not yet determined

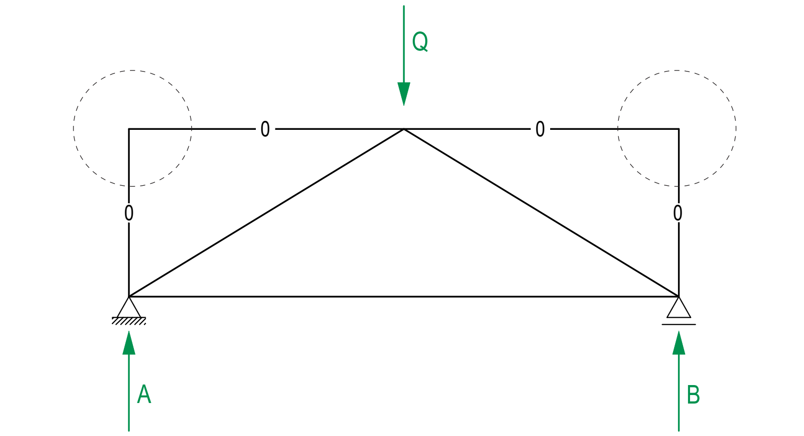


Zero Members

Zero members are elements, that do not take any load for a given loading condition. But in general, due to stability reasons they cannot be removed. Zero members can be distinguished while constructing the force diagram in nodes, where a local equilibrium cannot be created. In some cases, zero members can be identified beforehand. There are three rules for this purpose:

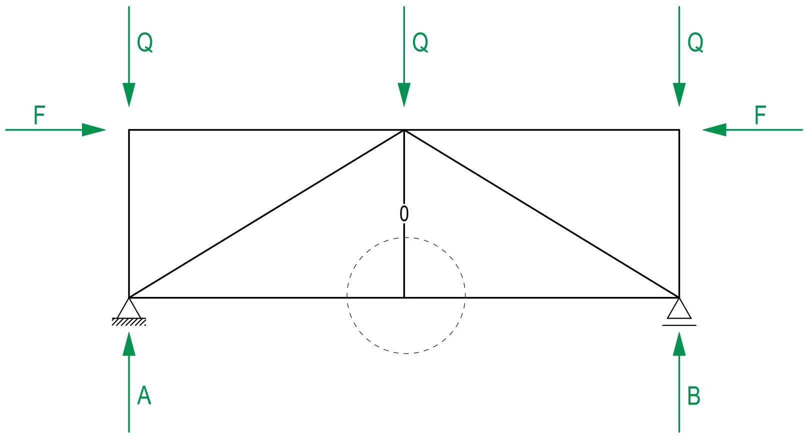
Rule 1

Unloaded node. If two bars with different directions meet, they are zero-members.



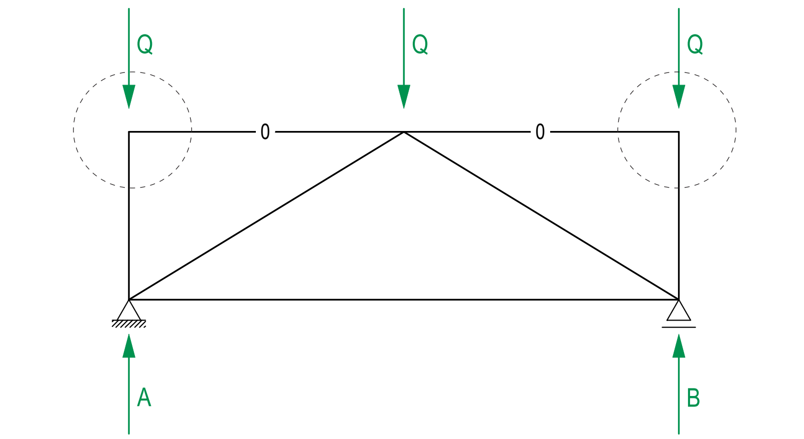
Rule 2

Unloaded node; three bars. If two bars have same direction, third one is a zero member.



Rule 3

Loaded node; two bars. If the load has the direction of one of the bars, the other one is a zero member.



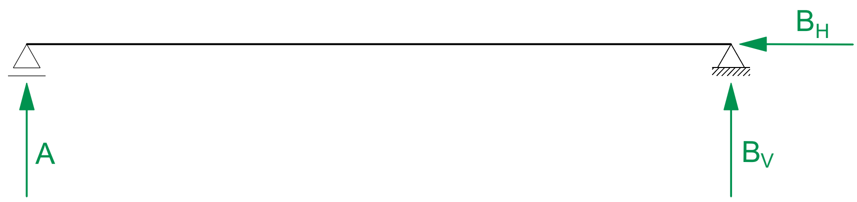
F13 Structural Design I
Statical Determinacy

The forces in the members of an ideal truss can be easily determined analytically or graphically if the truss is externally and internally statically determinate.

External Statical Determinacy

The external statical determinacy is a term for static systems and describes how many support reactions face the possible movements of the system.

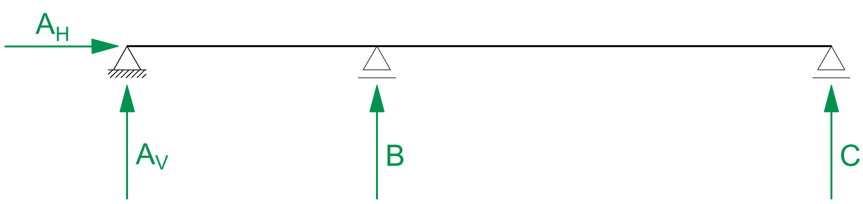
A system is externally statically determinate, if the number of support reactions is equal to the number of possible movement directions (equilibrium conditions). Since in a two-dimensional system there are three possible movement directions (horizontal, vertical and rotation), three support reactions are required for the system to be statically determinate and therefore stable.



3	Support Reactions	(A, B_V, B_H)
- 3	Equilibrium Conditions	($\sum V=0$, $\sum H=0$, $\sum M=0$)
0 =	statically determinate	

External Statical Indeterminacy

A system is externally statically indeterminate, if the number of support reactions exceeds the number of possible movement directions. In this case there are too many support reactions available. The degree of indeterminacy is the result of the difference between the number of support reactions and the number of movement directions. The calculation of such systems is performed using methods that go beyond fundamental equilibrium conditions.



4	Support Reactions	(A_H, A_V, B, C)
- 3	Equilibrium Conditions	($\sum V=0$, $\sum H=0$, $\sum M=0$)
1 =	onefold statically indeterminate	

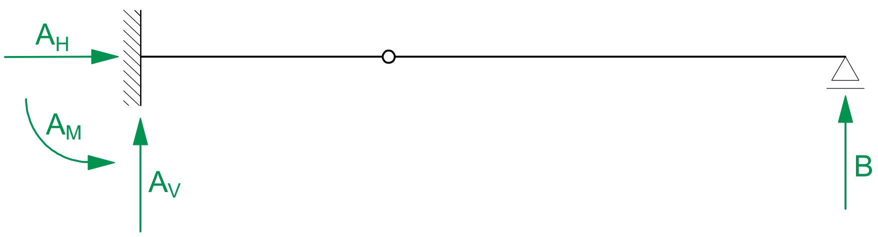
External Statical Underdeterminacy

A system is externally statically underdeterminate, if the number of support reactions is smaller than the number of possible movement directions. In this case the support reactions are insufficient. There are no solutions for the equations. There is no equilibrium, the system is instable.

2	Support Reactions	(A, B)
- 3	Equilibrium Conditions	($\sum$V=0, $\sum$H=0, $\sum$M=0)
- 1	= onefold statically underdeterminate	

Hinges as Additional Equilibrium Conditions

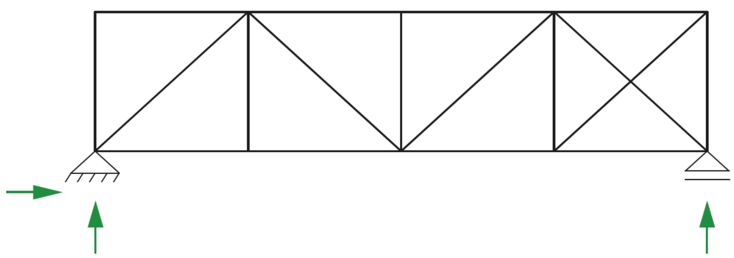
A hinge can not face a rotation. For this reason there is an additional equilibrium condition for hinges (M = 0). That means that one additional equilibrium condition can be introduced per hinge.



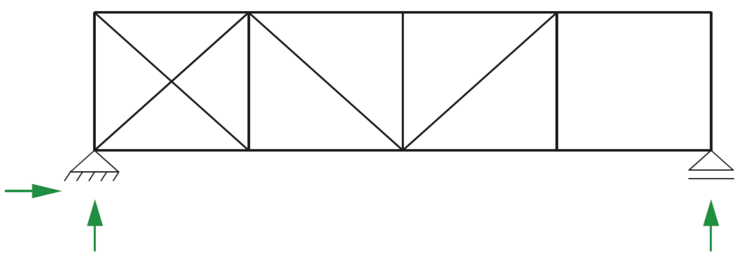
4	Support Reactions	(A_M, A_V, A_H, B)
- 1	Hinge	(M = 0)
- 3	Equilibrium Conditions	($\sum$V=0, $\sum$H=0, $\sum$M=0)
0	= statically determinate	

Internal Statical Determinacy in Trusses

A planar, externally statically determinate supported truss is also internally statically determinate, if the number of truss members corresponds to double the number of the nodes minus three. The number 3 stands in this equation for the number of equilibrium conditions (that corresponds to the number of support reactions in case of an externally statically determinate system). To shape an internally statically determinate truss, one should follow the triangle rule. Square meshes lead to unstable systems. The inner forces of truss members in statically indeterminate trusses can not be solved with simple methods like graphic statics.



Example: statically indeterminate truss (internally indeterminate, externally determinate static system). The introduction of an additional bar violates the shaping rule. So the truss is statically indeterminate.



Example: Unstable truss. Quadrangular meshes lead to unstable systems, even if the requirements for internal static determinacy are fulfilled.

Calculating the Statical Determinacy of Planar Trusses

S = number of truss members (bars)
K = number of nodes

$S = 2K - 3$ = statically determinate
$S < 2K - 3$ = unstable (underdeterminate)
$S > 2K - 3$ = statically indeterminate (overdeterminate)

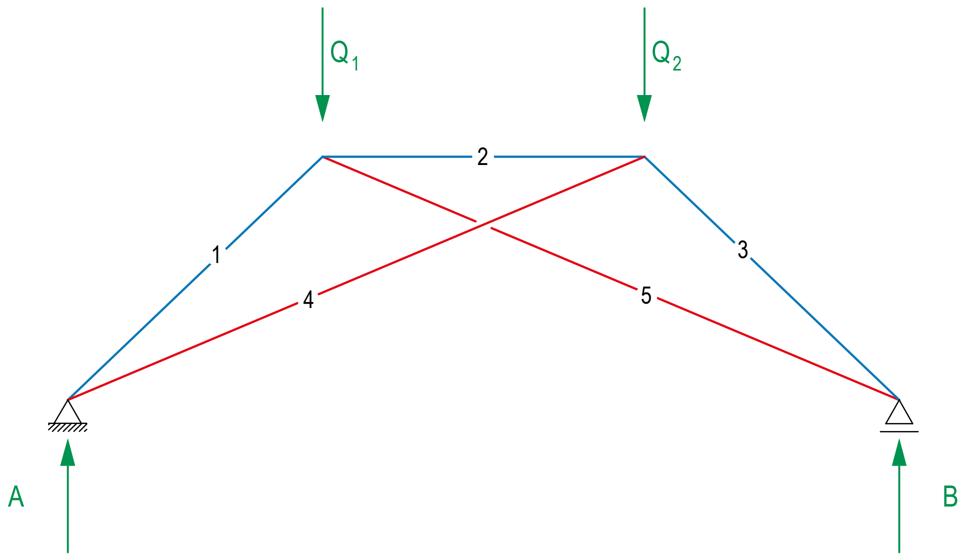
Calculation of inner statical determinacy of three-dimensional trusses
$S = 3K - 6$ = statically determinate

Intersecting Bars

In most cases in graphic statics there is one corresponding element in the force diagram per bar-element in the form diagram. This applies if there are no intersecting elements in the form diagram. If single bars intersect or overlap in the form diagram (as in the example below bars 4 and 5), they should be drawn in duplicate in the force diagram, so that the force polygons created can be closed.

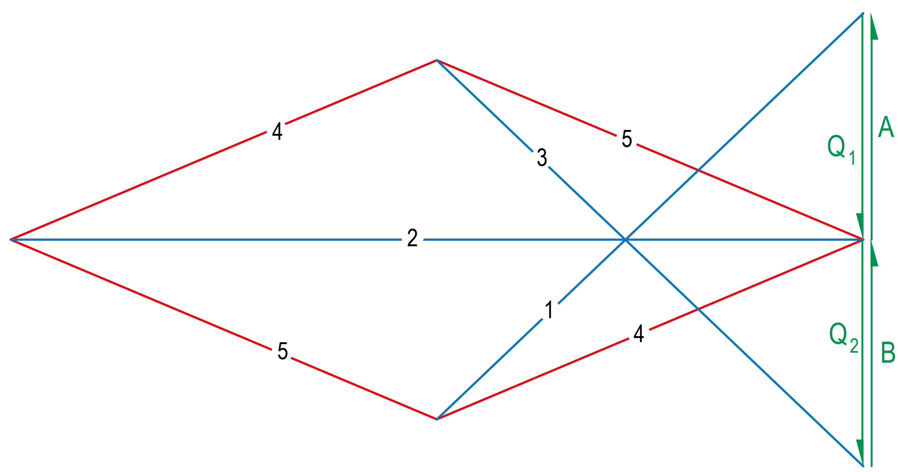
Form Diagram

Bar 4 and bar 5 are intersecting



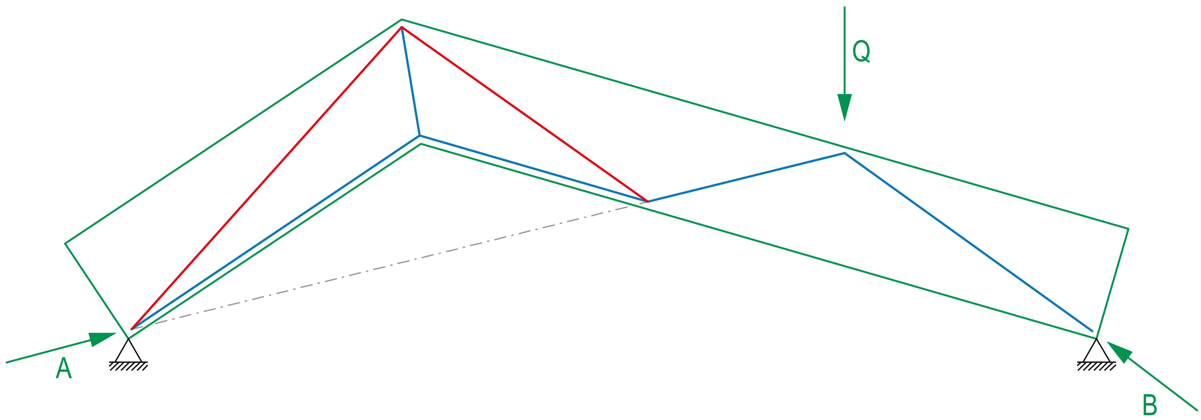
Force Diagram

Bar 4 and bar 5 appear twice



Redirecting Forces

To evolve possible redirections of the internal forces in beams, frames and walls, it is appropriate to use the direct distribution of the internal forces (thrust line/arch) as a starting point. In most cases, this thrust line is situated outside of the material. In order to redirect the internal forces, in addition to compression forces, tension forces need to be engaged. By means of these tension forces, the thrust line is redirected in a way, such that it remains inside the material.



The simplest way of redirecting is to shape the frame element, such that it consists of two cantilevers joined by one frame corner. The redirection shown below can be used for both compression and tension members.