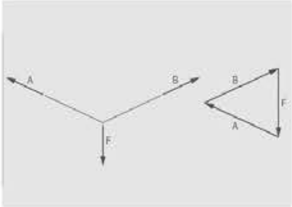
Tragwerksentwurf I

Structural Design I

Philippe Block · Joseph Schwartz


Tragwerksentwurf
Structural Design


1. Gleichgewicht
1. Equilibrium



2. Graphische Statik
2. Graphic statics

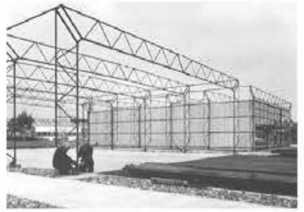

3. Seile
3. Cables


4. Bogen
4. Arches


5. Bogen-Seil
5. Arch-cables

Tragwerksentwurf I
Structural Design I

Tragwerksentwurf II
Structural Design II


6. Fachwerke
6. Trusses

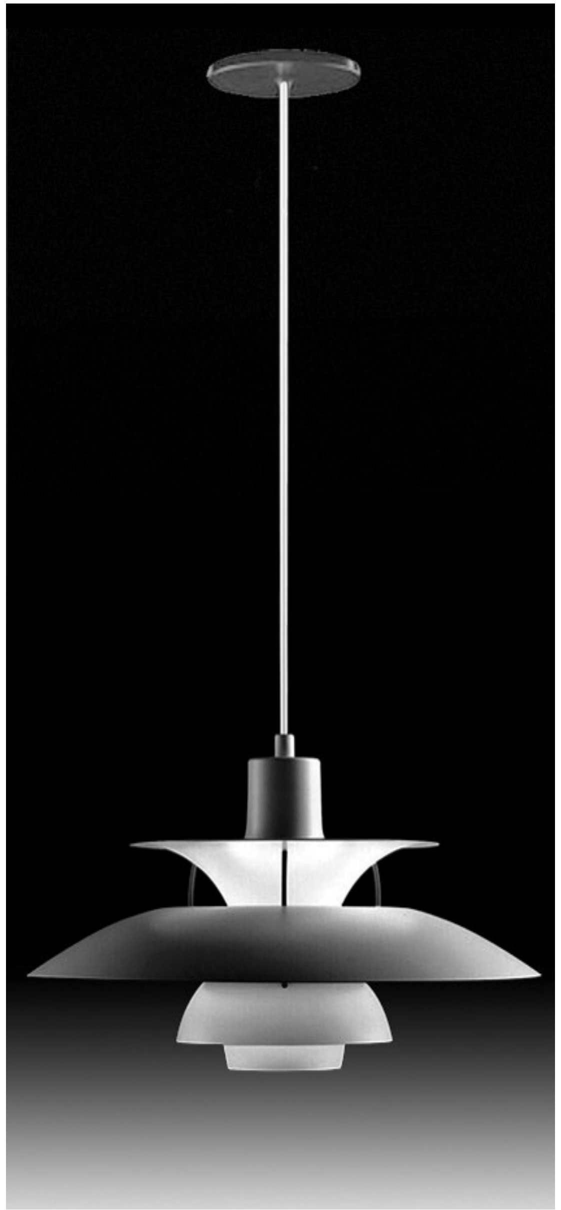

7. Balken
7. Beams

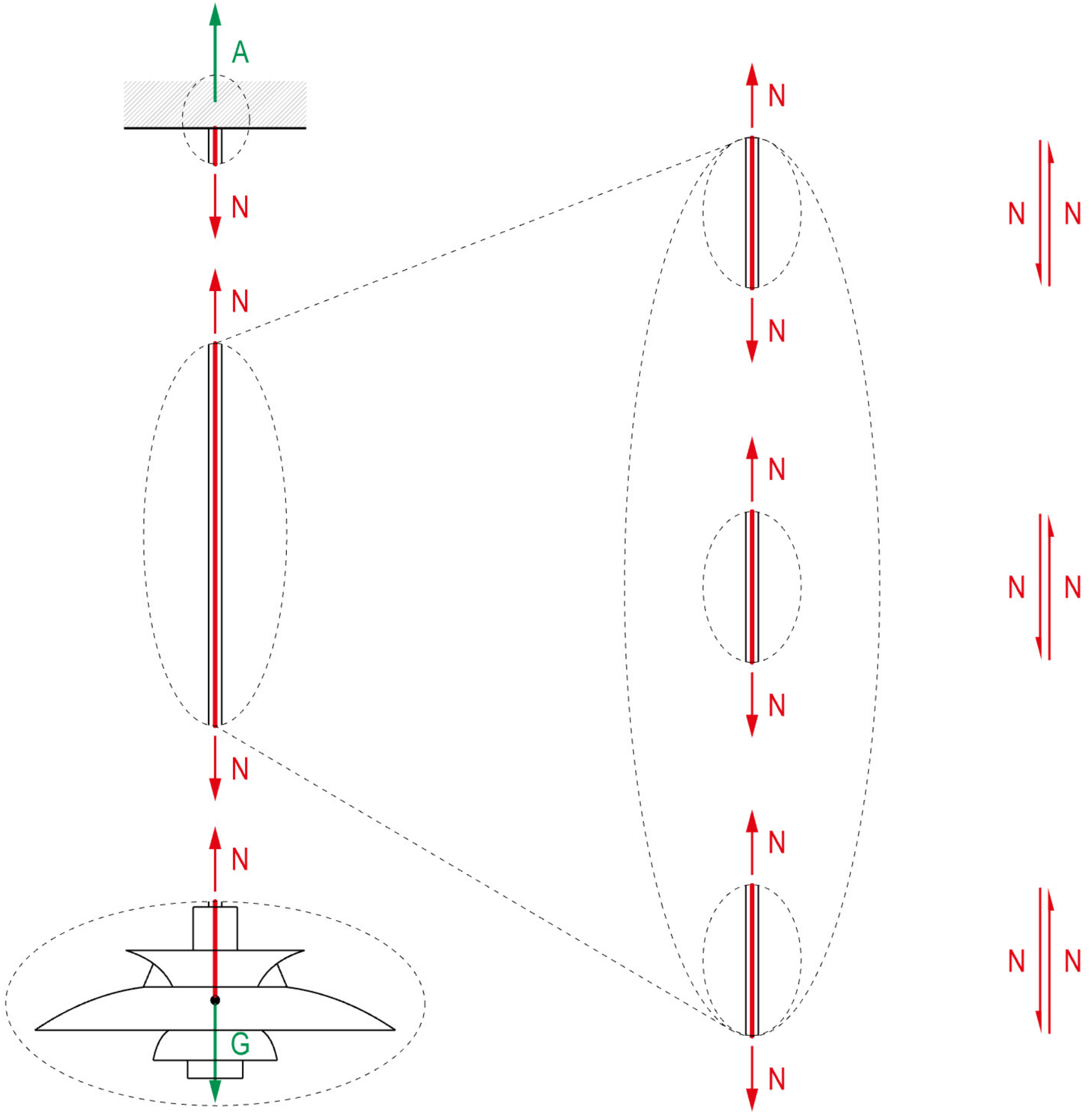

8. Rahmen
8. Frames


9. Platten
9. Plates


10. Stützen
10. Columns


11. Nachhaltigkeit
11. Sustainability





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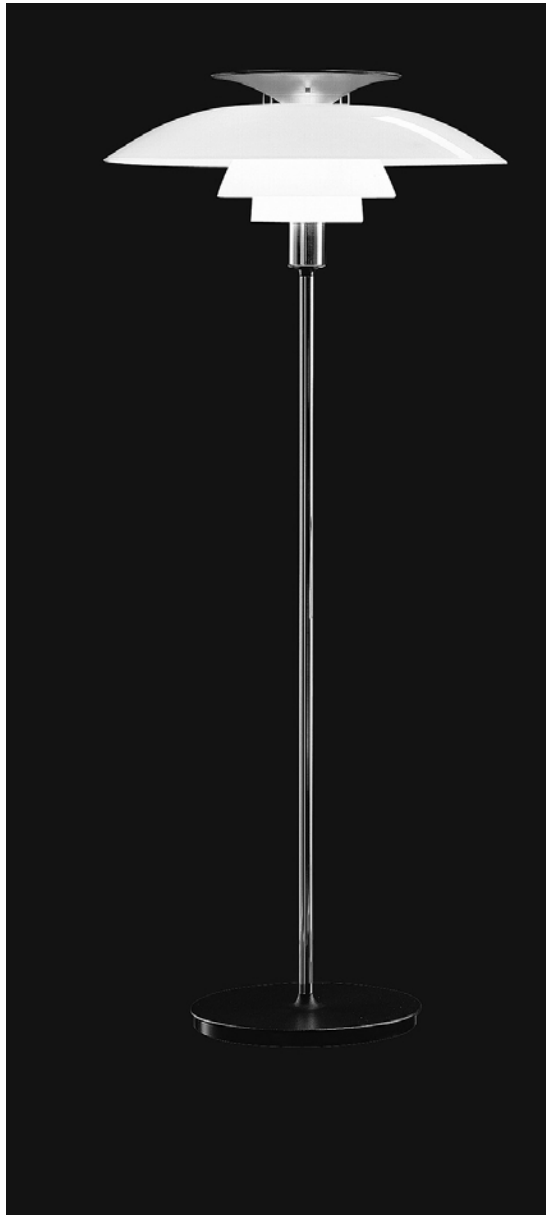

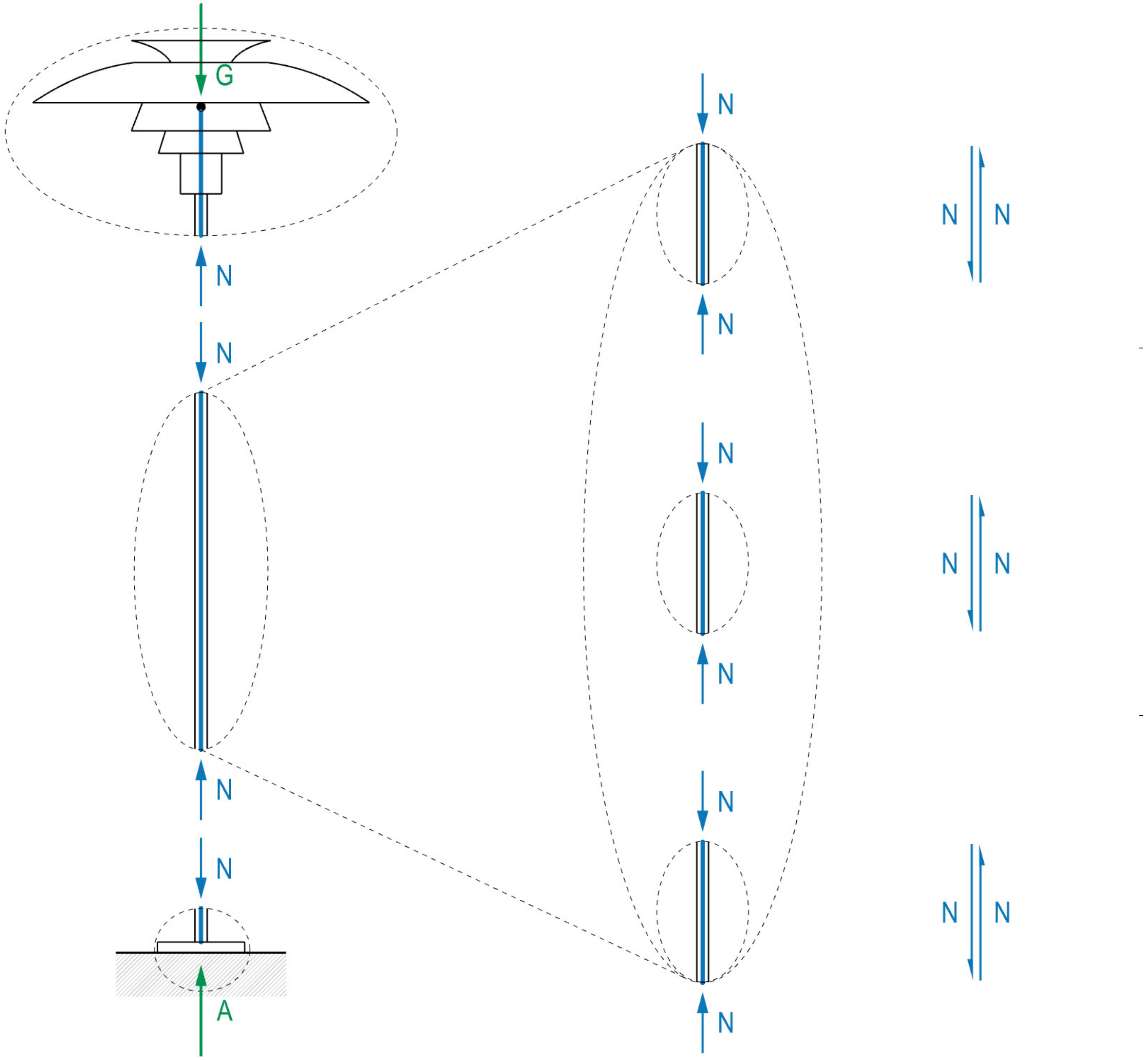
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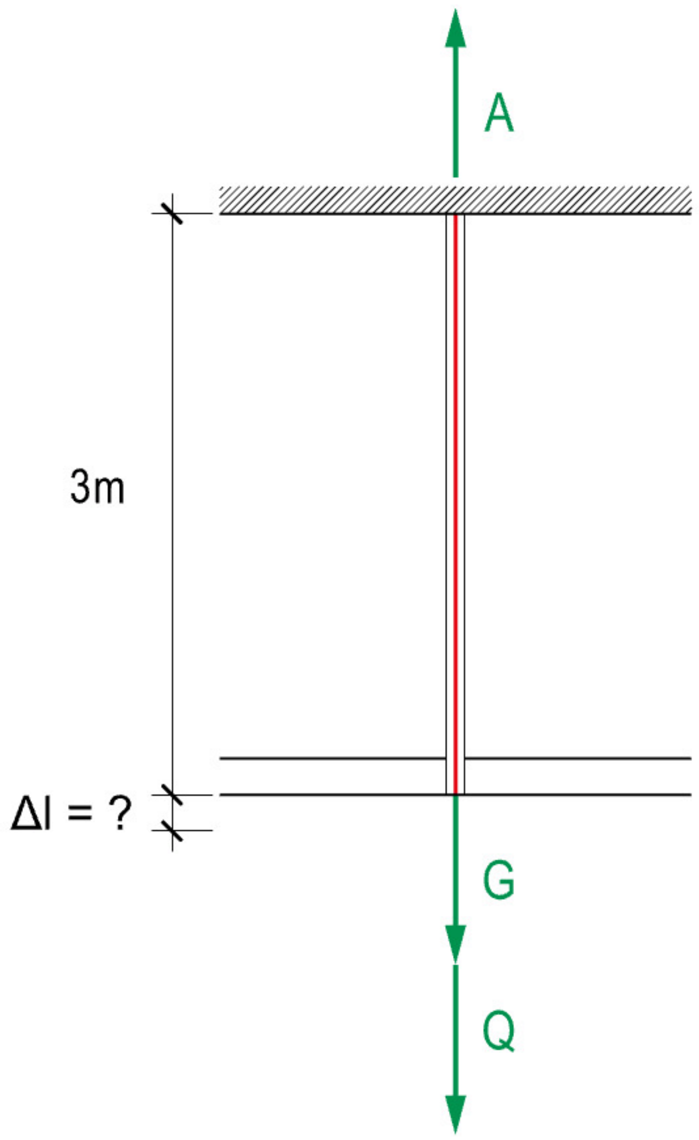


Ziel der Bemessung ist
The goal of dimensioning is

- den Bruch zu vermeiden (Tragsicherheit)
  to prevent material failure (Ultimate Limit State)

- die Verformung zu begrenzen (Gebrauchstauglichkeit)
  to limit the deformation (Serviceability Limit State)

Material und Bemessung
Material and dimensioning

>> Innere Kräfte und Verformung im Tragelement
Internal forces and deformation in a structural element

Steifigkeit des Tragelements
Stiffness of a structural element

Innere Kräfte im Material
Internal forces in the material

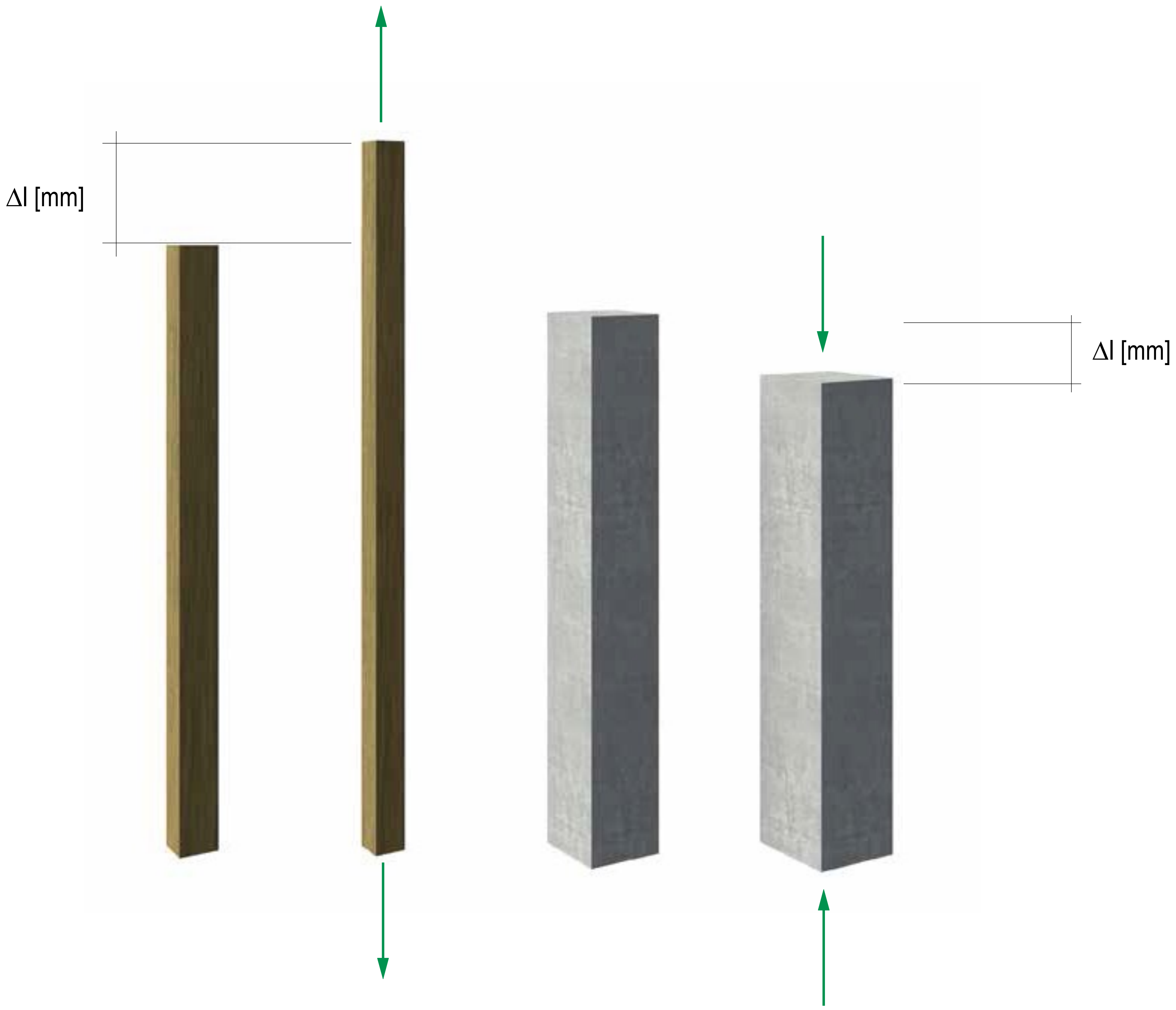
Verformung und Steifigkeit des Materials
Deformation and stiffness of the material

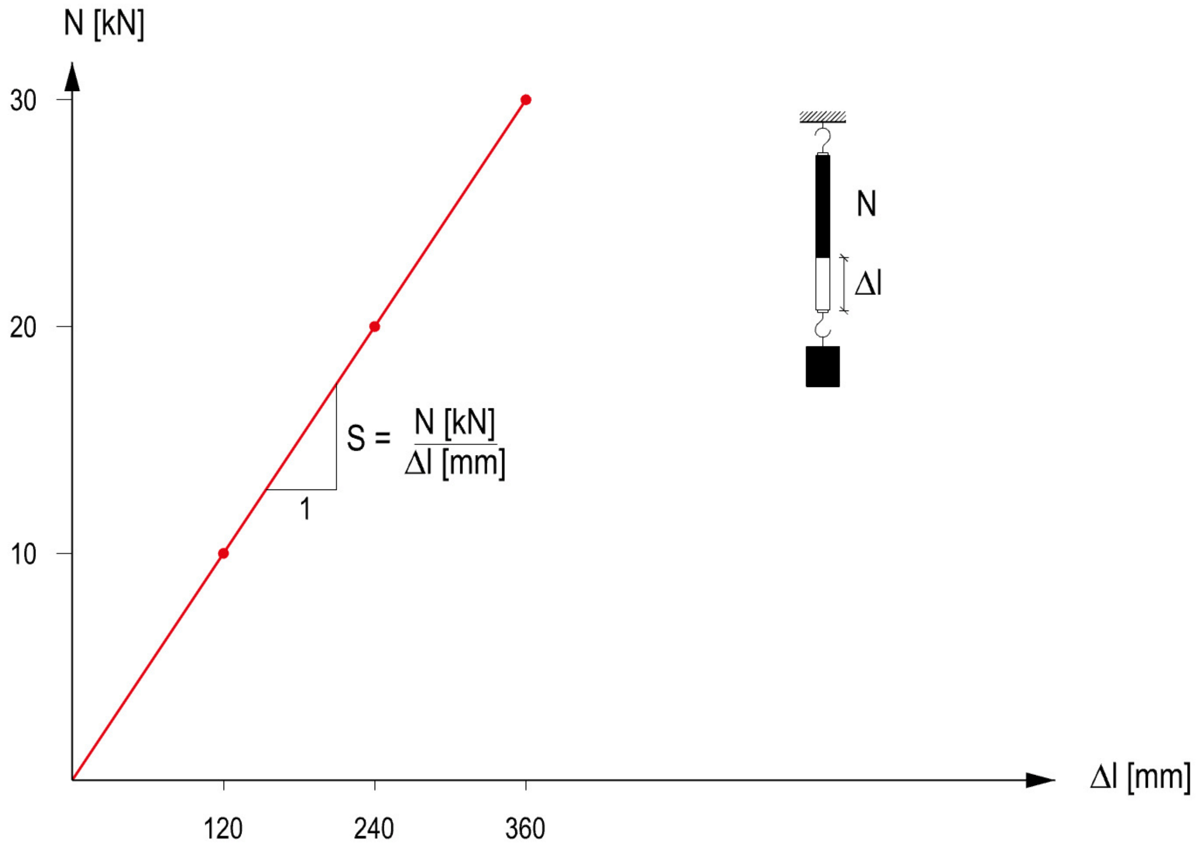
Materialverhalten verschiedener Baustoffe
Behaviour of different building materials

Bemessung für Grenzzustände
Dimensioning for limit states

Verformung aufgrund Temperaturveränderung
Deformation caused by temperature changes

Ermüdung
Fatigue


Δl [mm]
Δl [mm]


N [kN]
30
20
10
120
240
360
Δl [mm]
S = N [kN] / Δl [mm]
1
N
Δl

Material und Bemessung

Material and dimensioning

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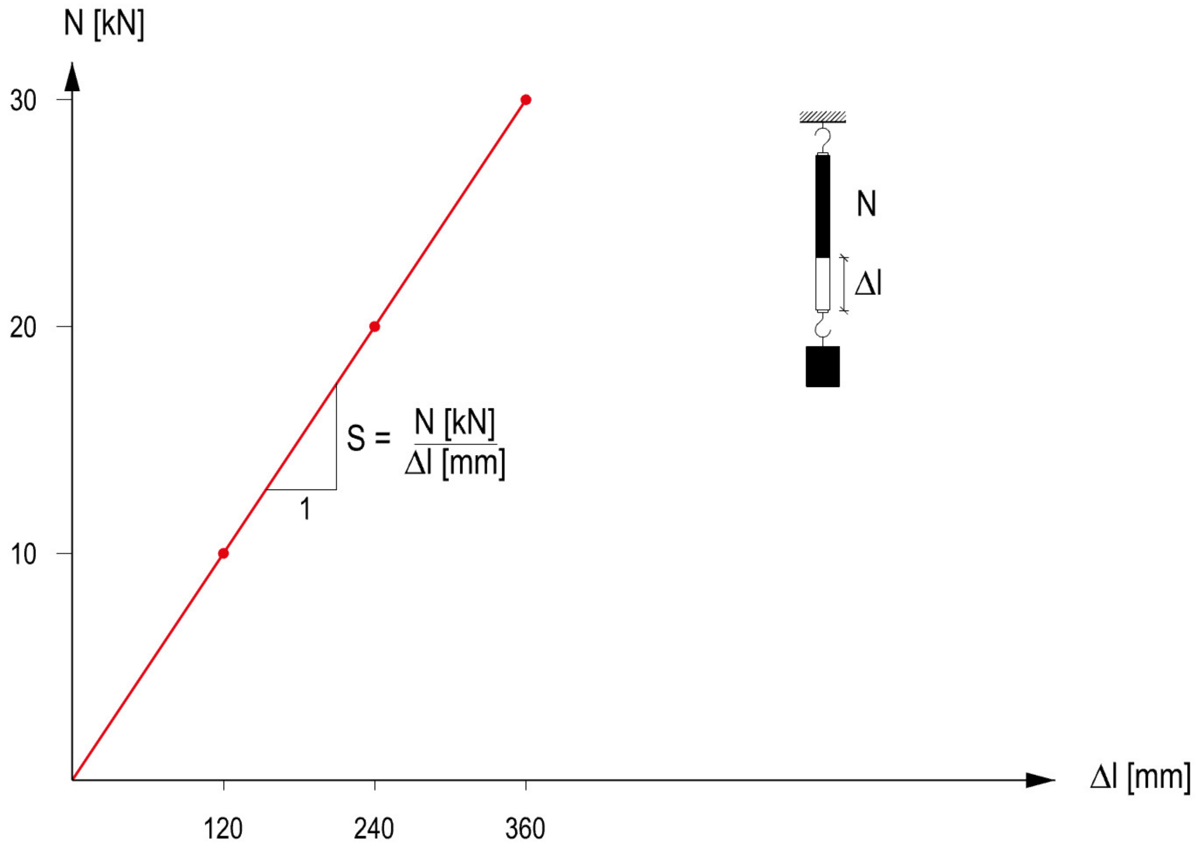
N [kN]
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20
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S = N [kN] / Δl [mm]
1
120
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Δl [mm]
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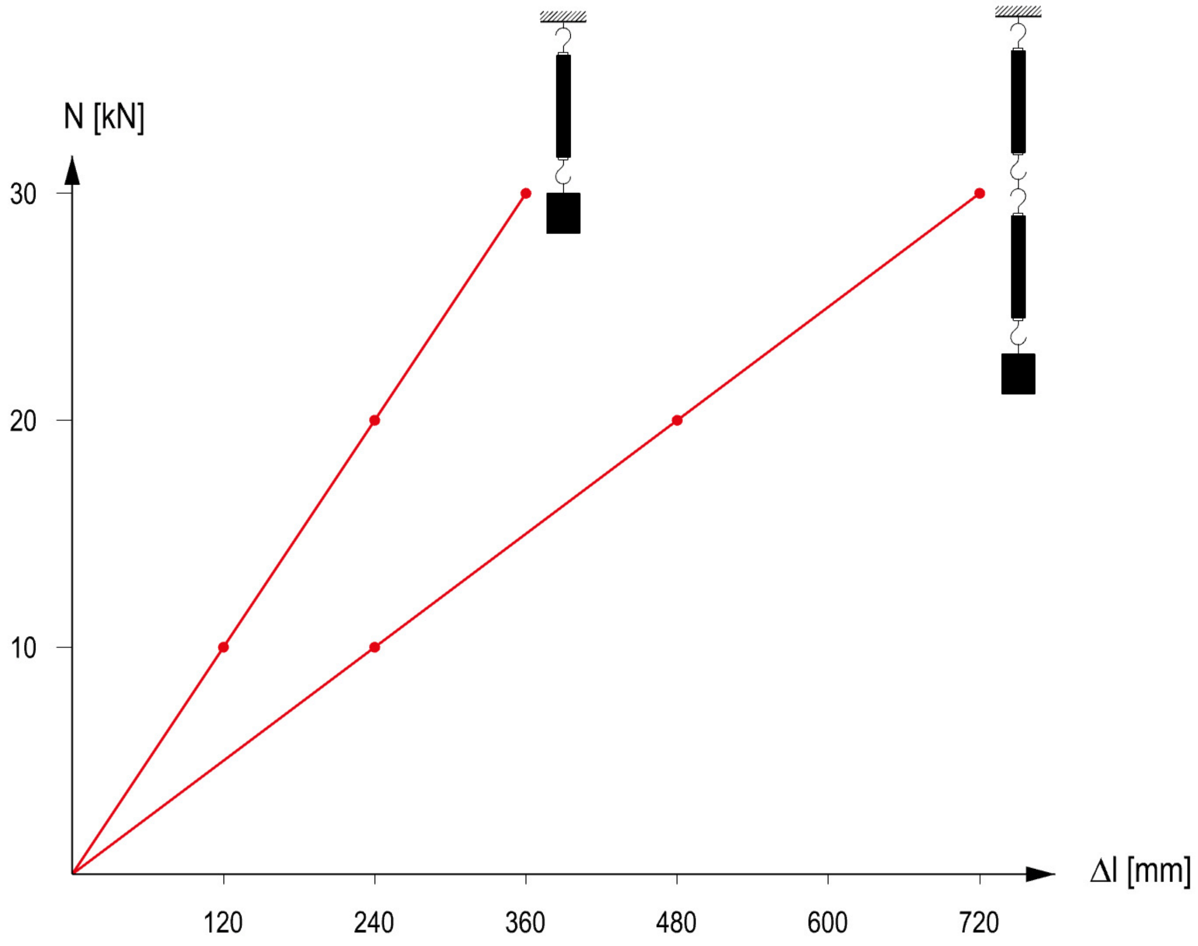
Δl


N [kN]
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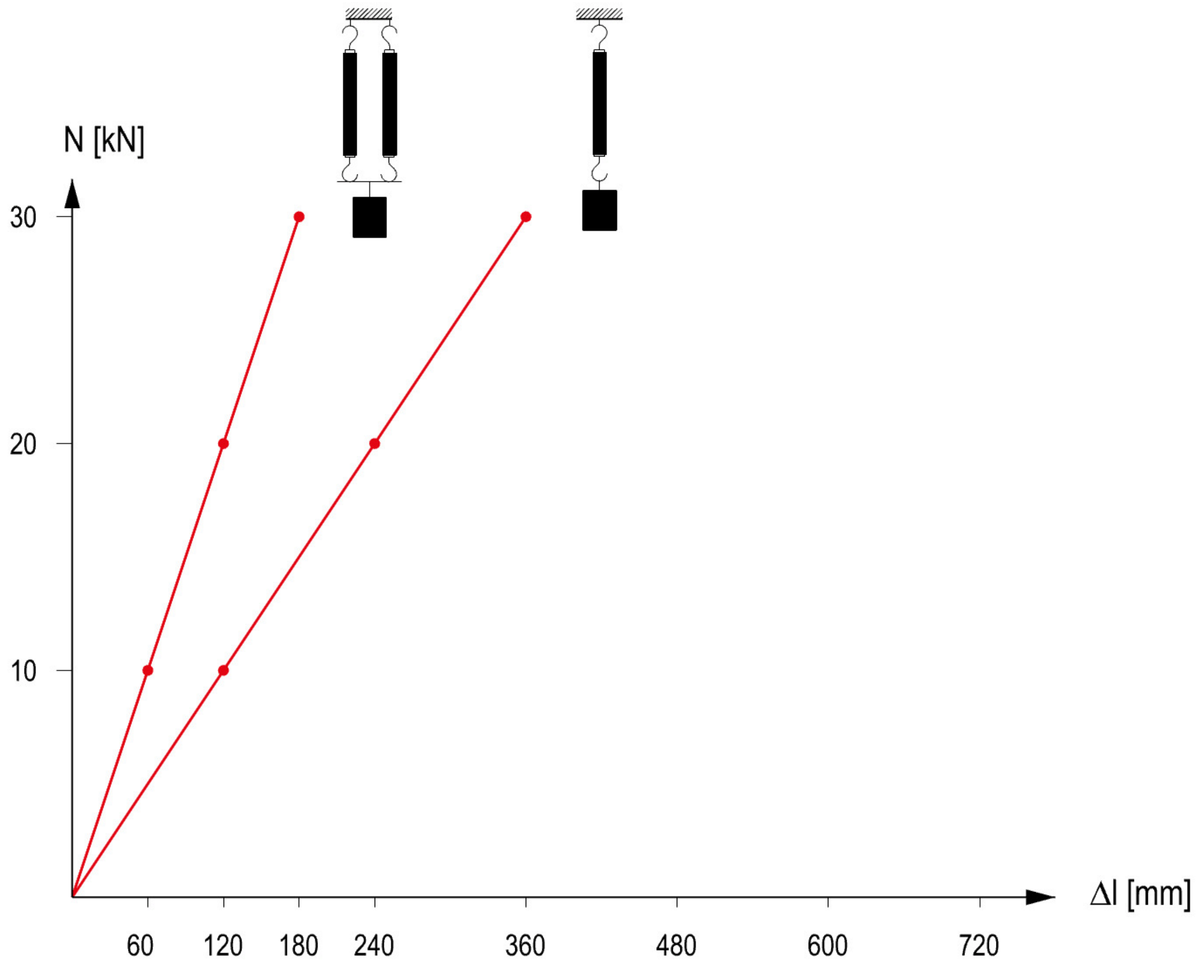
10
Δl [mm]
120
240
360
480
600
720


N [kN]
30
20
10
Δl [mm]
60
120
180
240
360
480
600
720

$$S = \frac{N}{\Delta l} = E * \frac{A}{l}$$

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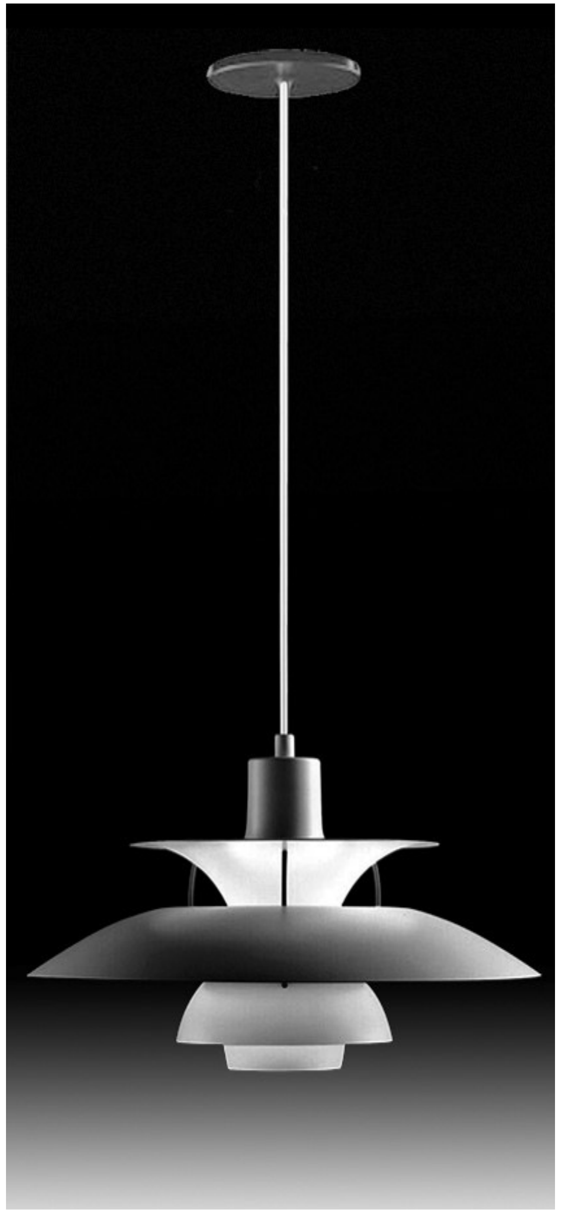

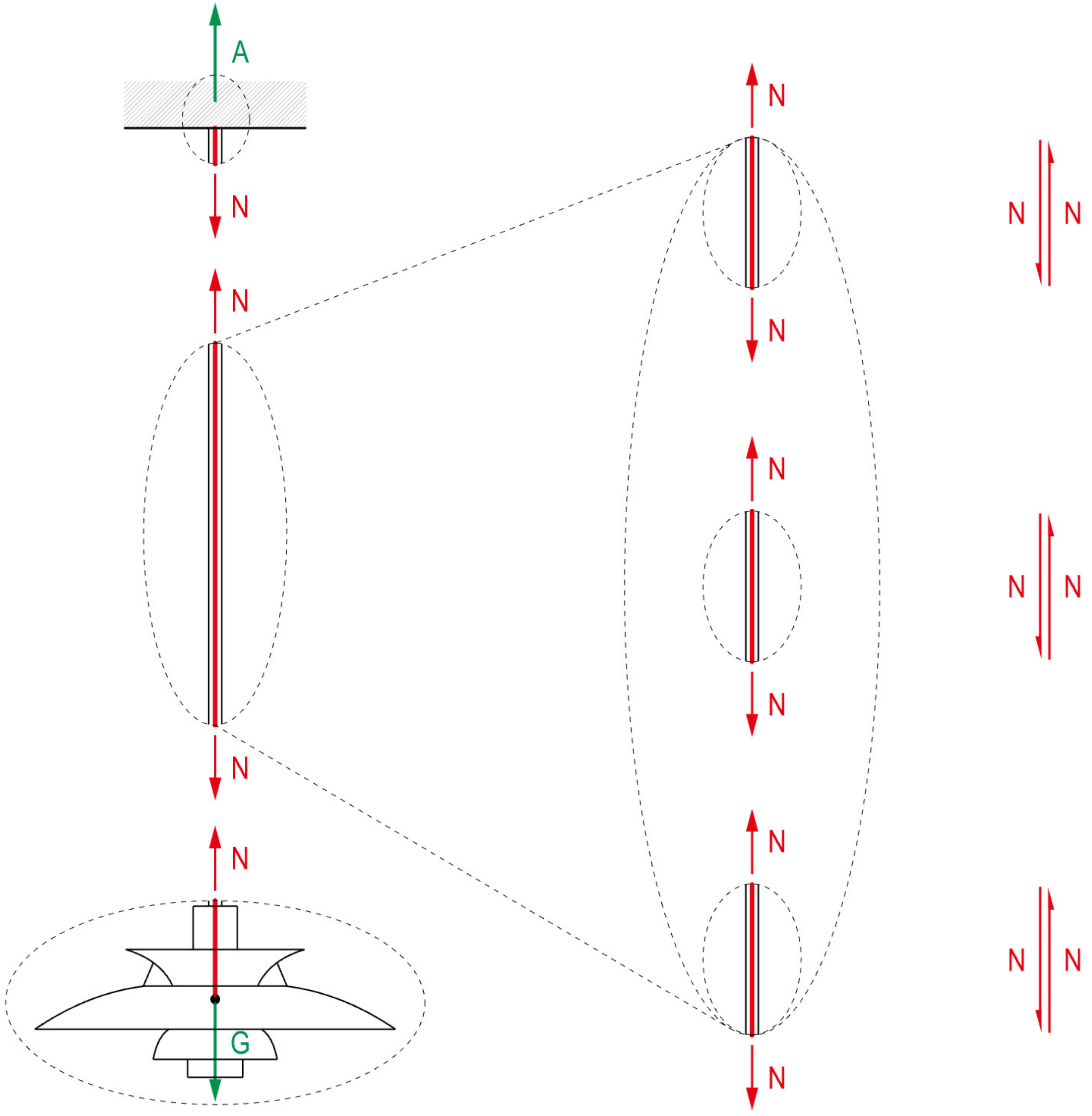
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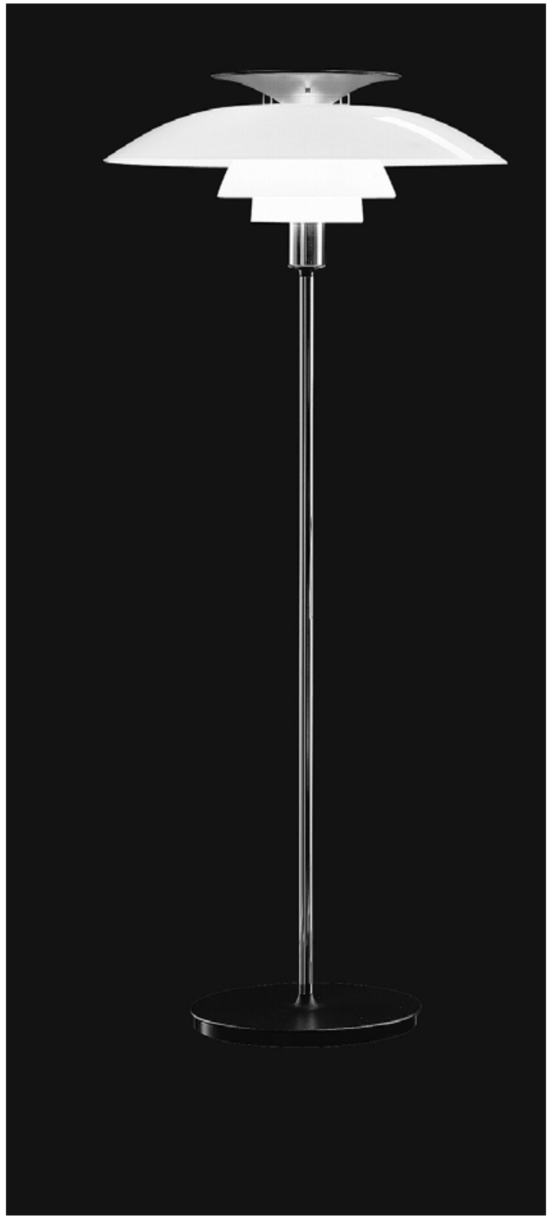

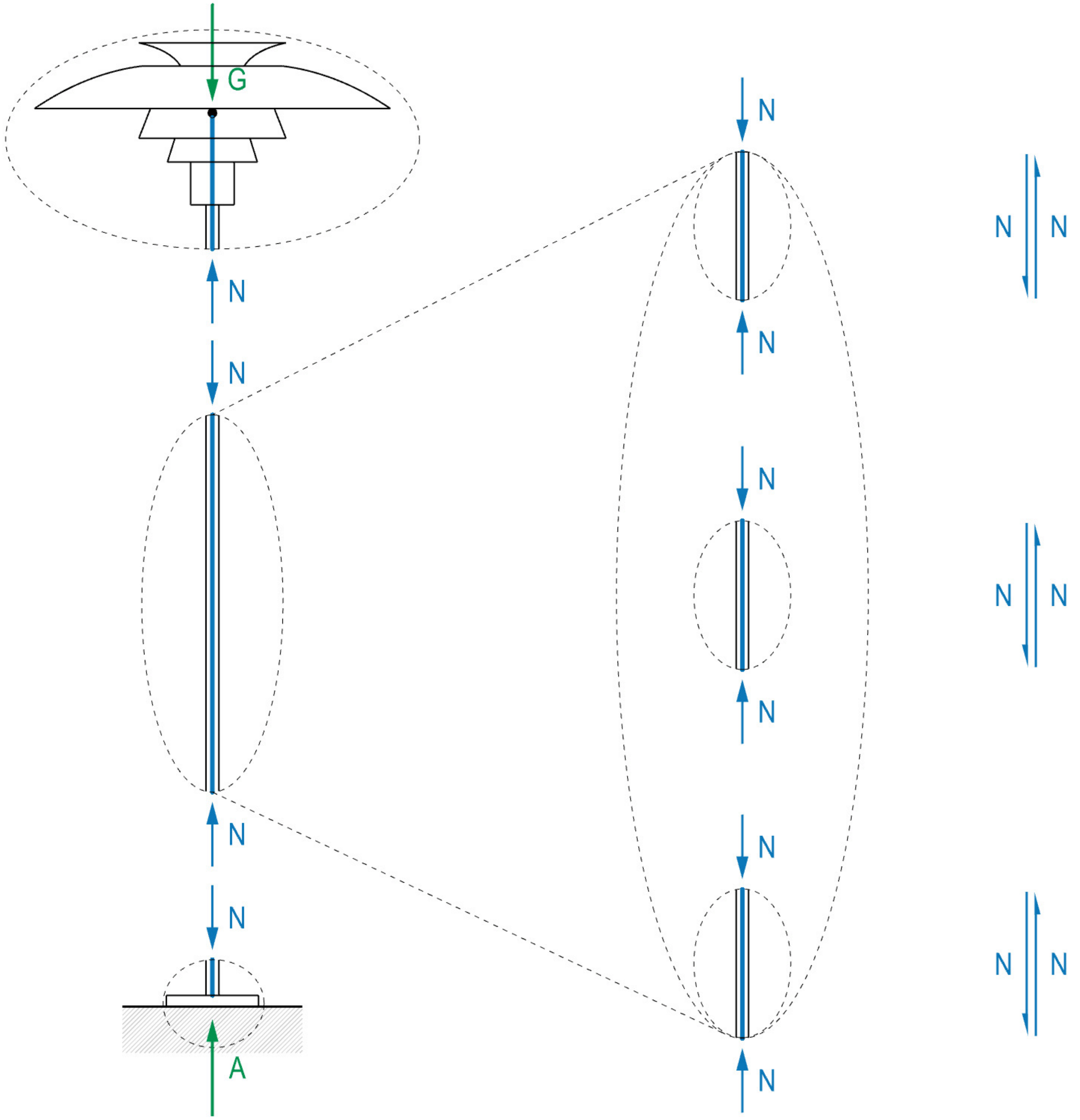
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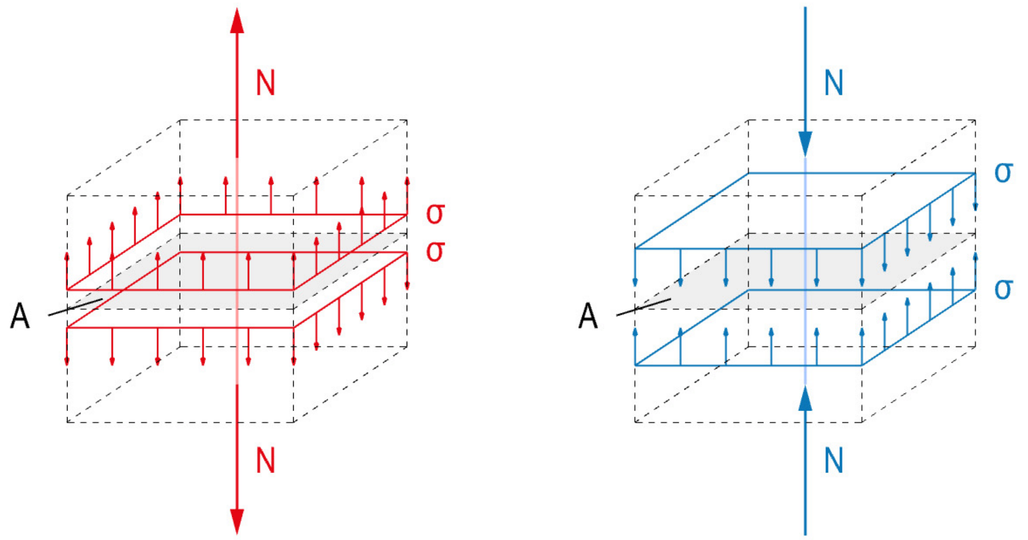

$$\sigma = \frac{N}{A}$$

Spannung

Stress

Material und Bemessung

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Steifigkeit
Stiffness

$$S = \frac{N}{\Delta l} = E * \frac{A}{l}$$

$$\frac{N}{A} = E * \frac{\Delta l}{l}$$

Spannung
Stress

$$\sigma = \frac{N}{A}$$

$$\varepsilon = \frac{\Delta l}{l}$$

Dehnung
Strain

$$\sigma = E * \varepsilon$$

„Ceiiinosssttuv“

„Ut tensio sic vis“

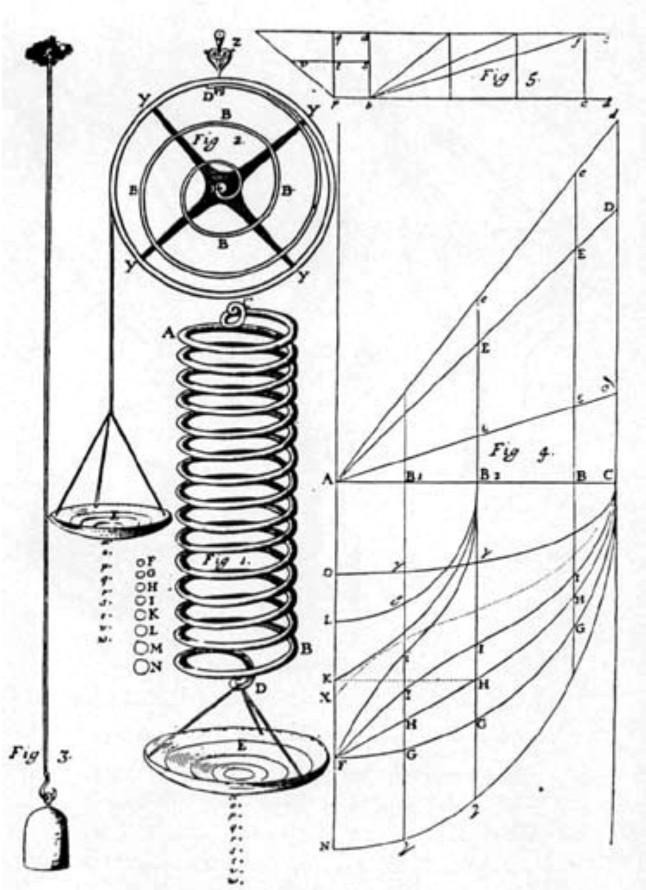
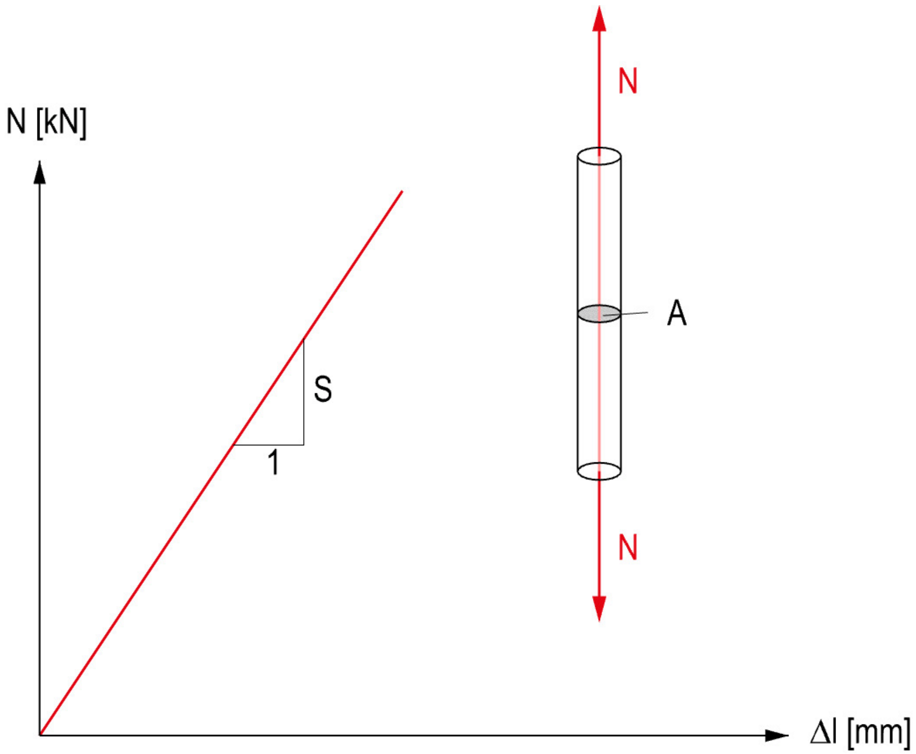


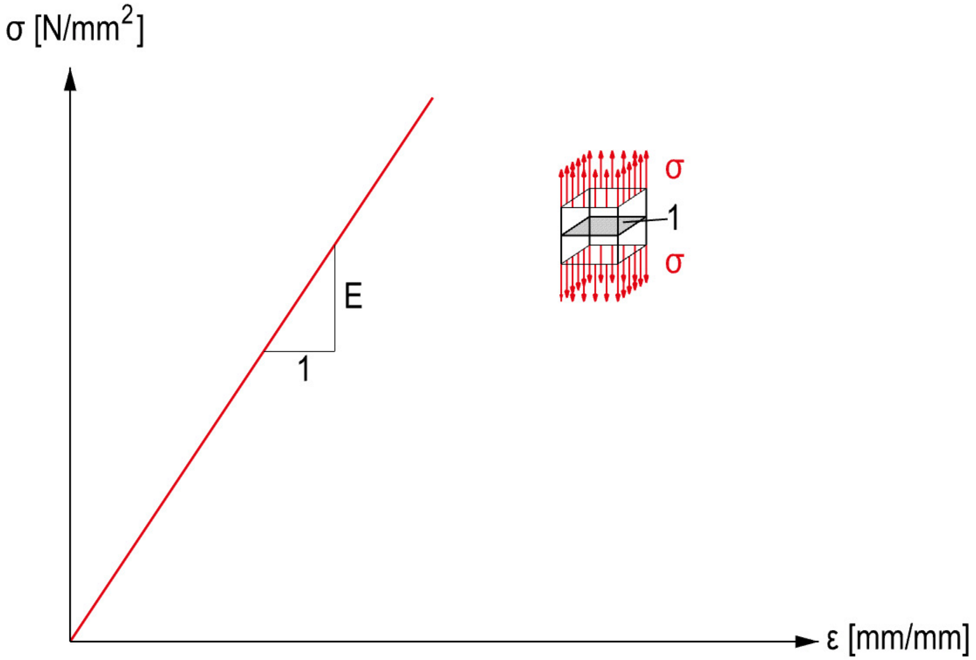
PLATE TO HOOKE'S LECTURE 'OF SPRING' 1678.

FIG. 1. Wire helical spring stretched to points *s*, *p*, *q*, *r*, *s*, *t*, *v*, *w*, by weights F, G, H, I, K, L, M, N.
FIG. 2. Watch spring similarly stretched by weights put in pan.
FIG. 3. The 'Springing of a string of Brass Wire 36 ft. long'.
FIG. 4. Diagram of velocities of springs.
FIG. 5. Diagram of law of ascent and descent of heavy bodies.

S = Steifigkeit des Tragelements
S = Stiffness of a structural element



E = Steifigkeit des Materials
E = Stiffness of the material



Material und Bemessung

Material and dimensioning

Innere Kräfte und Verformung im Tragelement
Internal forces and deformation in a structural element

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Stiffness of a structural element

Innere Kräfte im Material
Internal forces in the material

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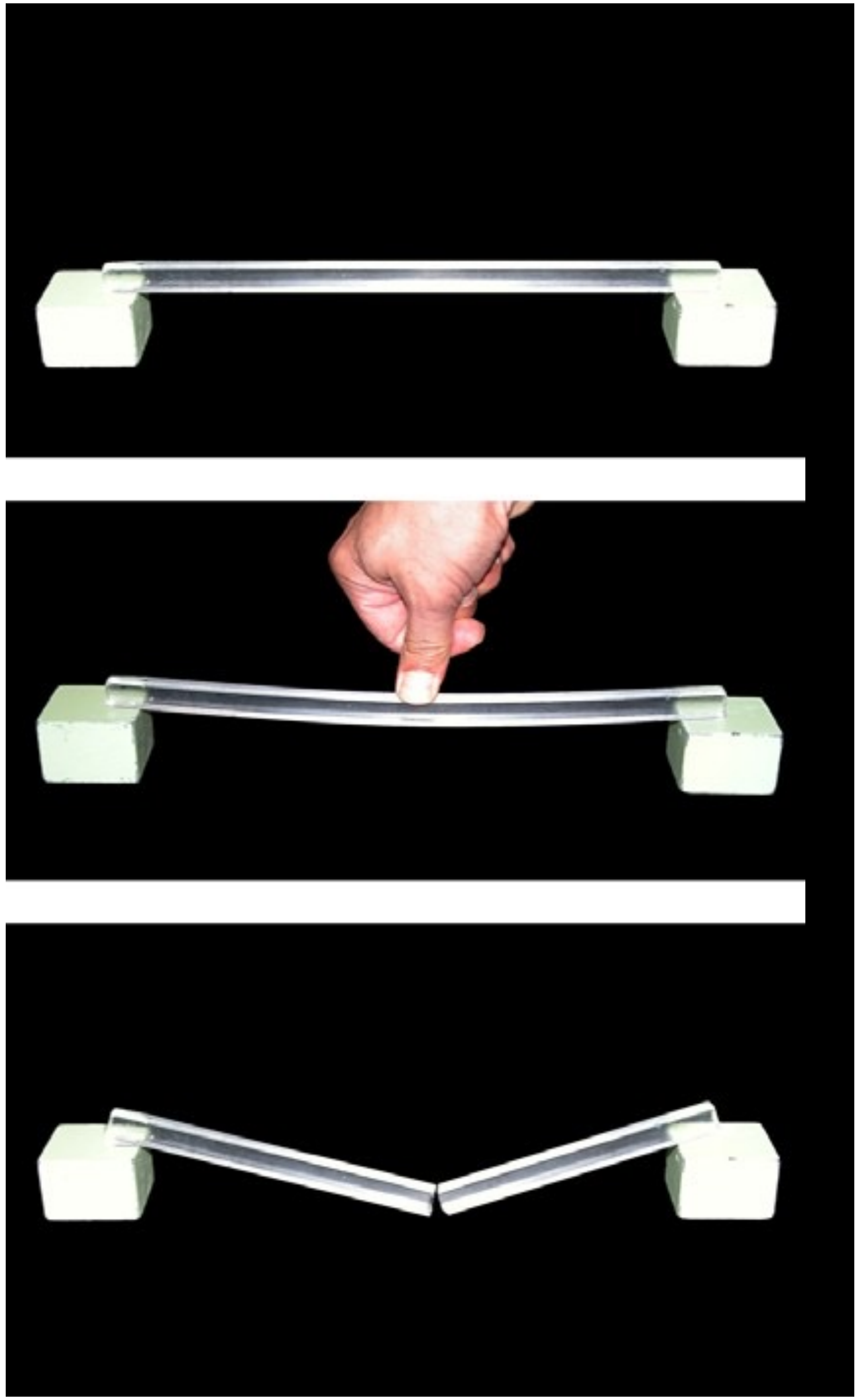
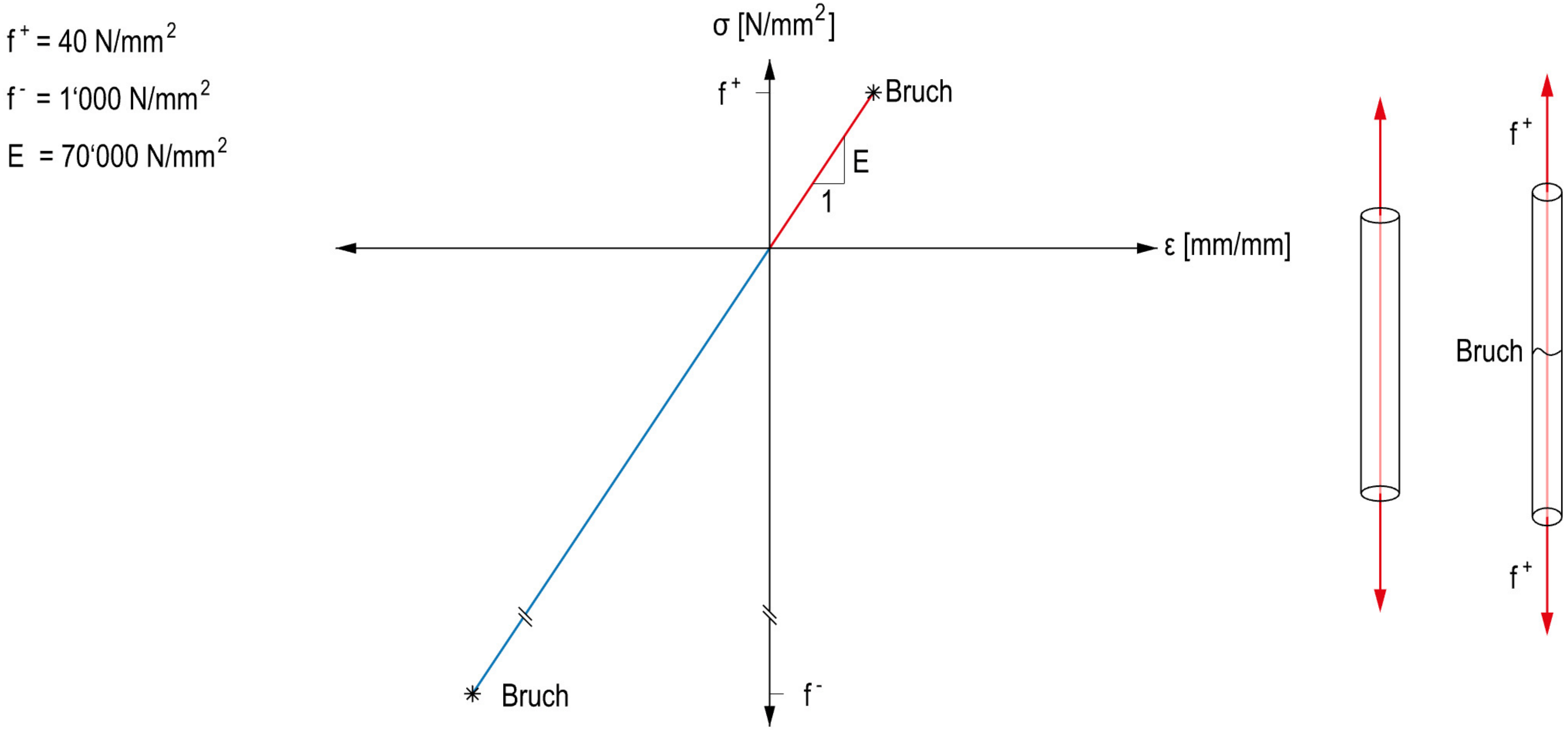
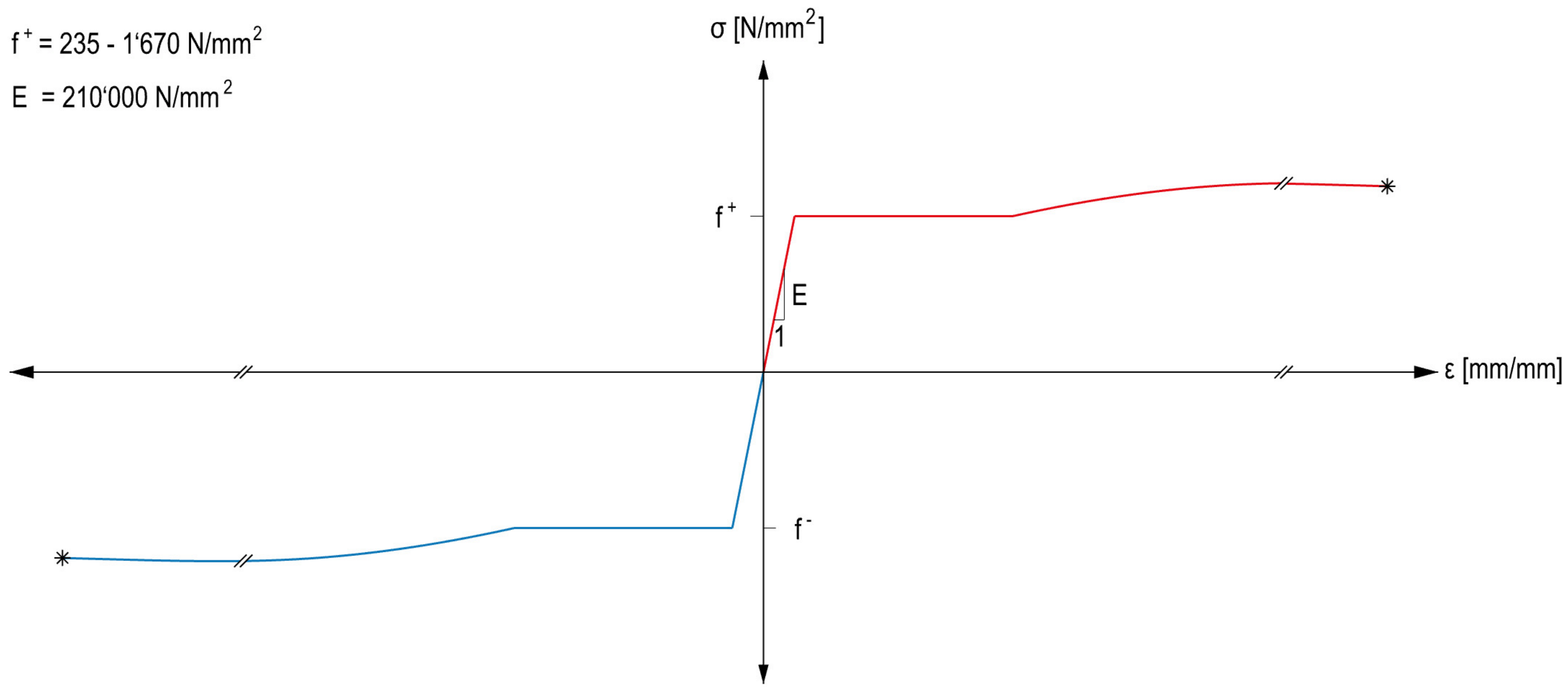
Fatigue





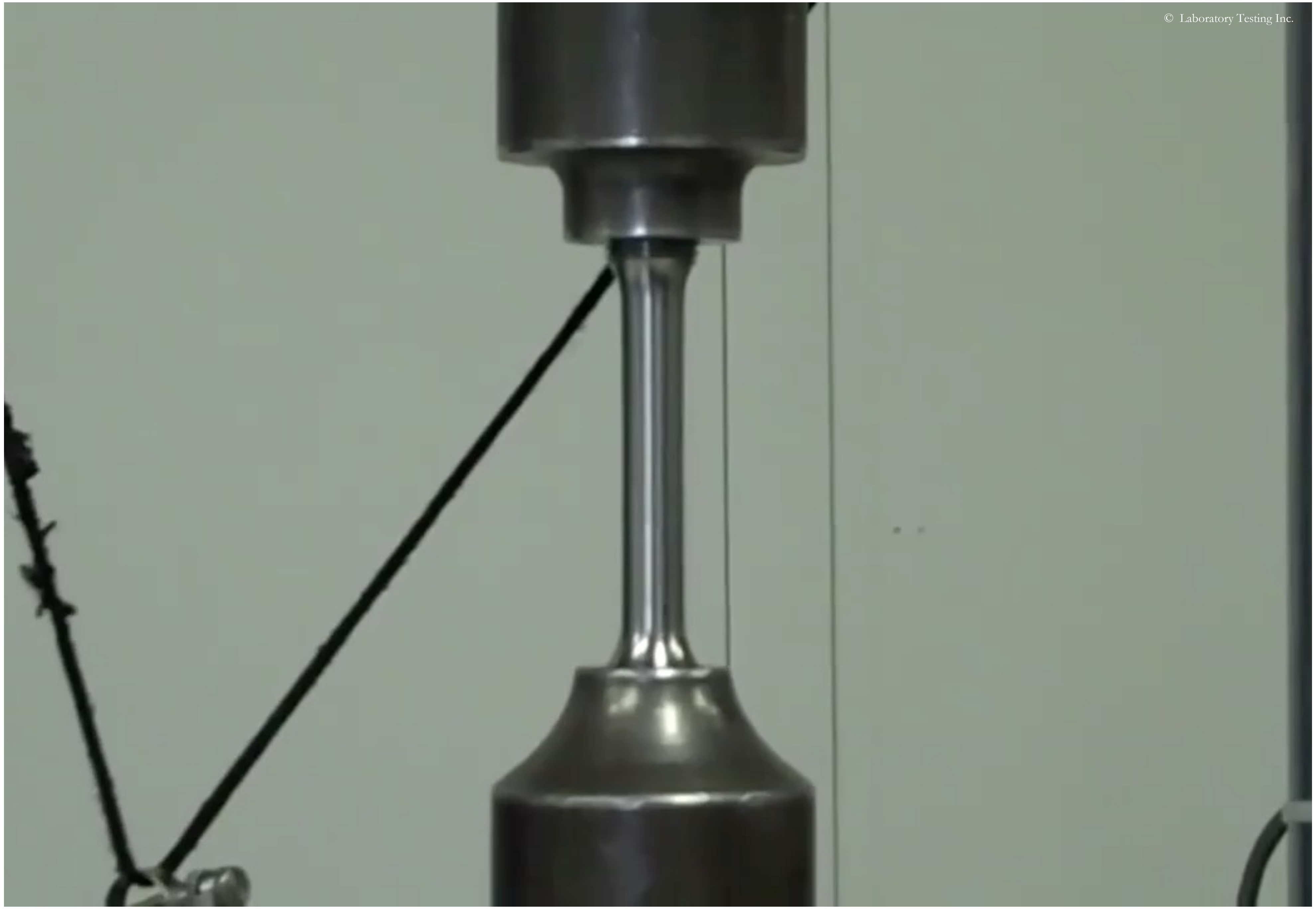
f^+ Zugfestigkeit
Tensile strength
f^- Druckfestigkeit
Compressive strength
E Elastizitätsmodul
Modulus of elasticity

Materialverhalten von Glas
Material behaviour of glass



f + Fliessgrenze
Yield strength
E Elastizitätsmodul
Modulus of elasticity

Materialverhalten von Stahl
Material behaviour of steel

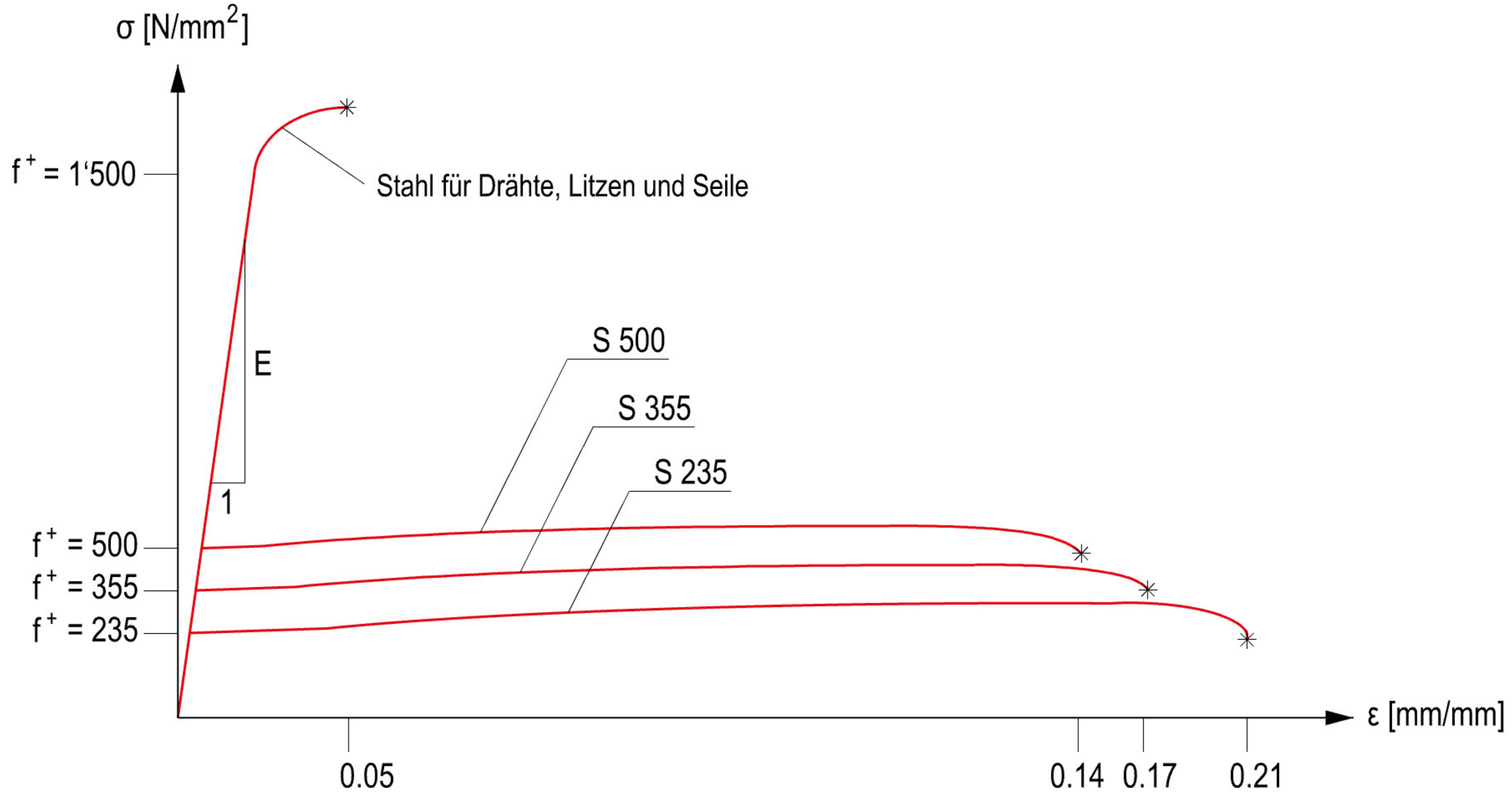


Zugversuch

Tensile test

f^+ = 235 - 1'670 N/mm^2

E = 210'000 N/mm^2

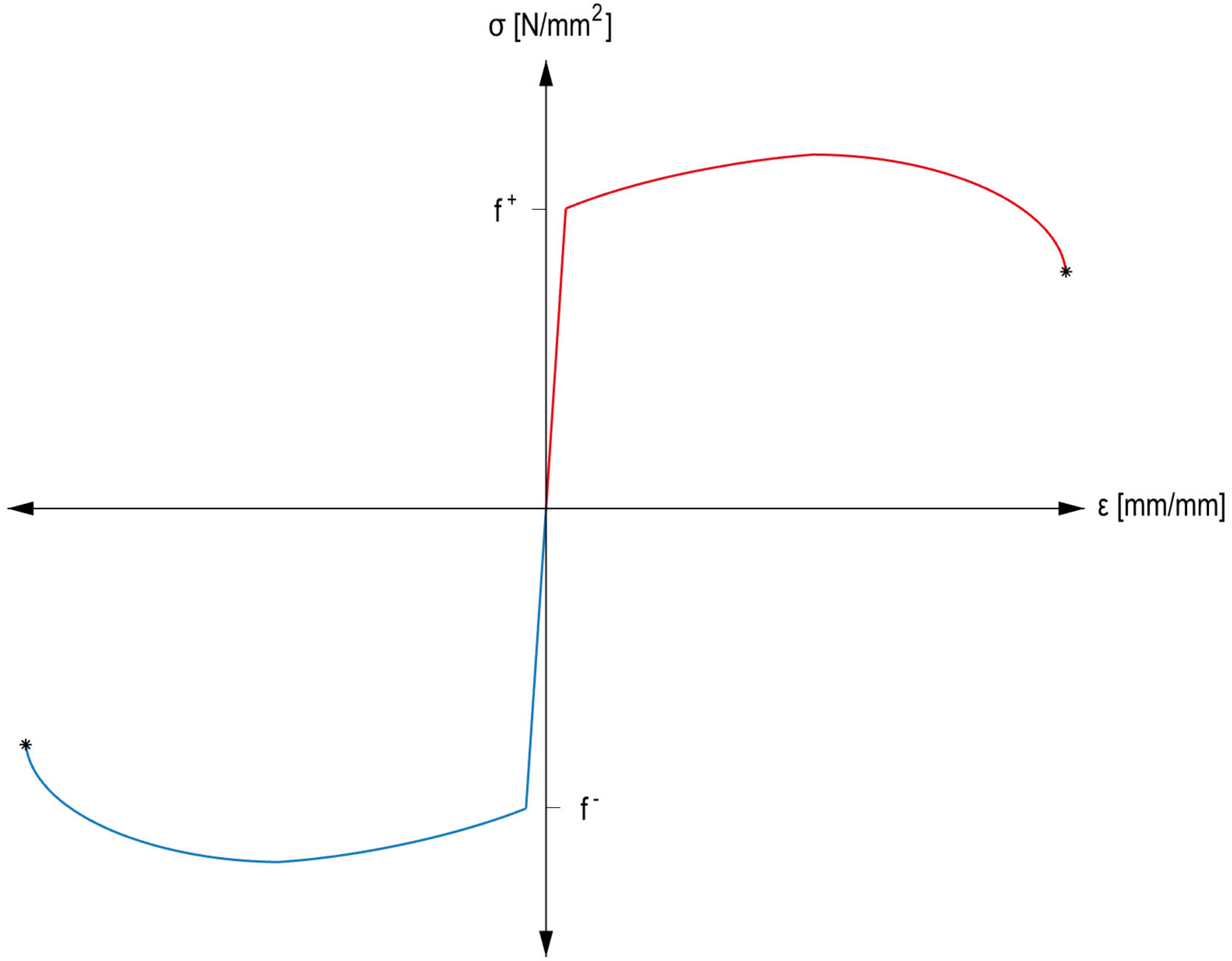


f^+ Fliessgrenze
Yield strength
E Elastizitätsmodul
Modulus of elasticity

Materialverhalten von Stahl
Material behaviour of steel

f^{+} = 180 - 300 N/mm^2

E = 70'000 N/mm^2



f^{+} Fliessgrenze
Yield strength

E Elastizitätsmodul
Modulus of elasticity

Materialverhalten von Aluminium
Material behaviour of aluminium

Beton mit geringer Festigkeit
Low-strength concrete

$f^+ = 1.1 - 2.0\ N/mm^2$

$f^- = 15\ N/mm^2$

$E = 28'000 - 34'000\ N/mm^2$

Im Bauwesen verwendeter Beton
Concrete used in construction

$f^+ = 1.5 - 2.9\ N/mm^2$

$f^- = 25\ N/mm^2$

$E = 30'000 - 36'000\ N/mm^2$

Beton für Brücken
Concrete for bridges

$f^+ = 2.2 - 4.2\ N/mm^2$

$f^- = 35\ N/mm^2$

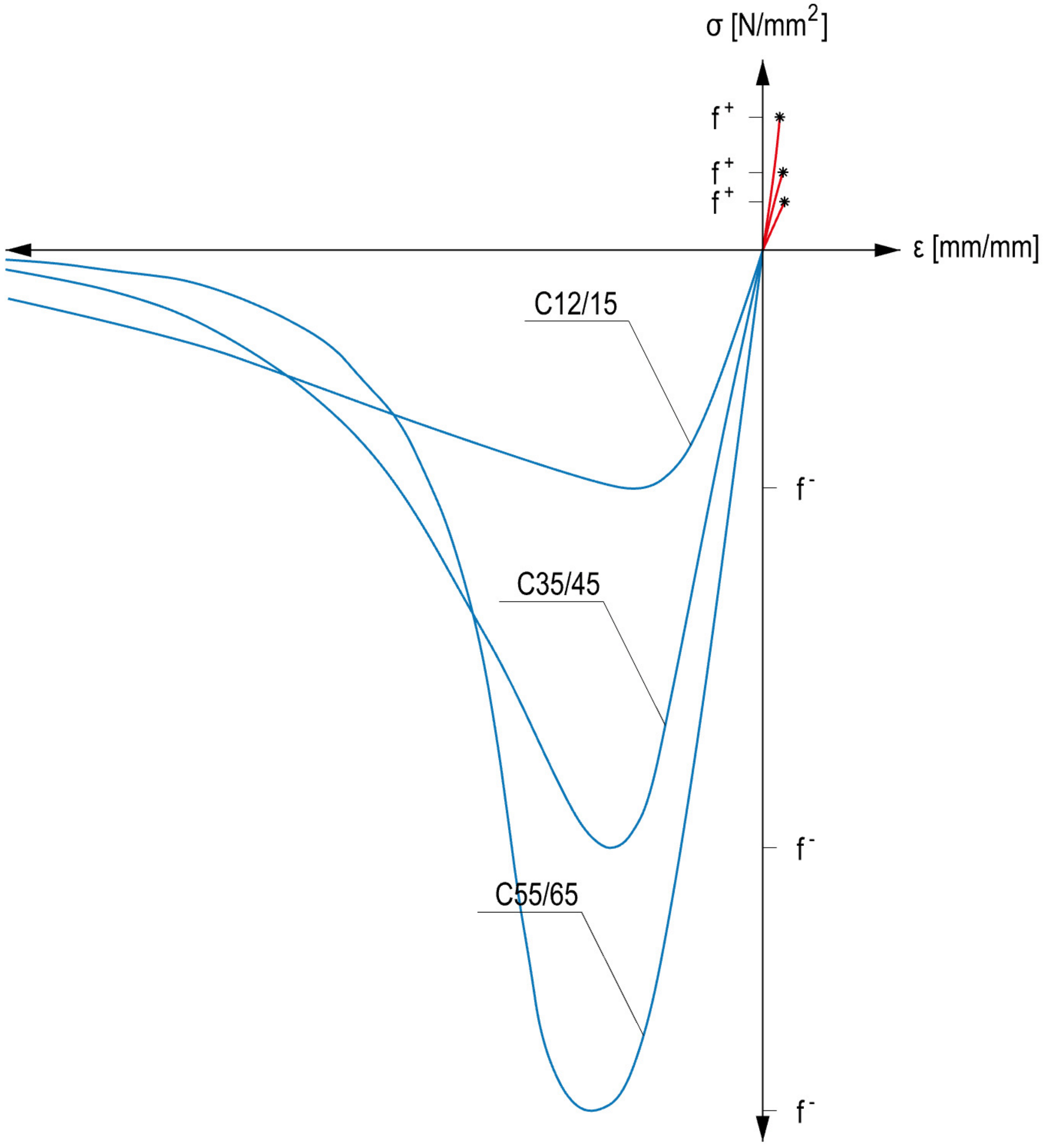
$E = 34'000 - 42'000\ N/mm^2$

Beton mit hoher Festigkeit
High-strength concrete

$f^+ = 2.9 - 5.3\ N/mm^2$

$f^- = 60\ N/mm^2$

$E = 37'000 - 44'000\ N/mm^2$



f^+ Zugfestigkeit
Tensile strength
f^- Druckfestigkeit
Compressive strength
E Elastizitätsmodul
Modulus of elasticity

Materialverhalten von Beton
Material behaviour of concrete

Materialverhalten von Beton

Material behaviour of concrete

	Stahl S500 *Steel S500*	Beton C20/25 *Concrete C20/25*	Verhältnis Stahl / Beton *Relation steel/concrete*
Elastizitätsmodul *Modulus of Elasticity*	210‘000 N/mm²	~33‘000 N/mm²	~6:1
Zugfestigkeit *Tensile strength*	580 N/mm²	~2.2 N/mm²	~300:1
Bruchdehnung *Strain*	140mm/m	0.06mm/m	~2‘000:1
Rohdichte *Density*	7‘850 kg/m³	2‘500 kg/m³	~3:1

Stahl und Beton im Vergleich
Steel and concrete in relation

Kalkstein
Sandstone

$f^{+} = 1 - 2\ N/mm^2$

$f^{-} = 10 - 60\ N/mm^2$

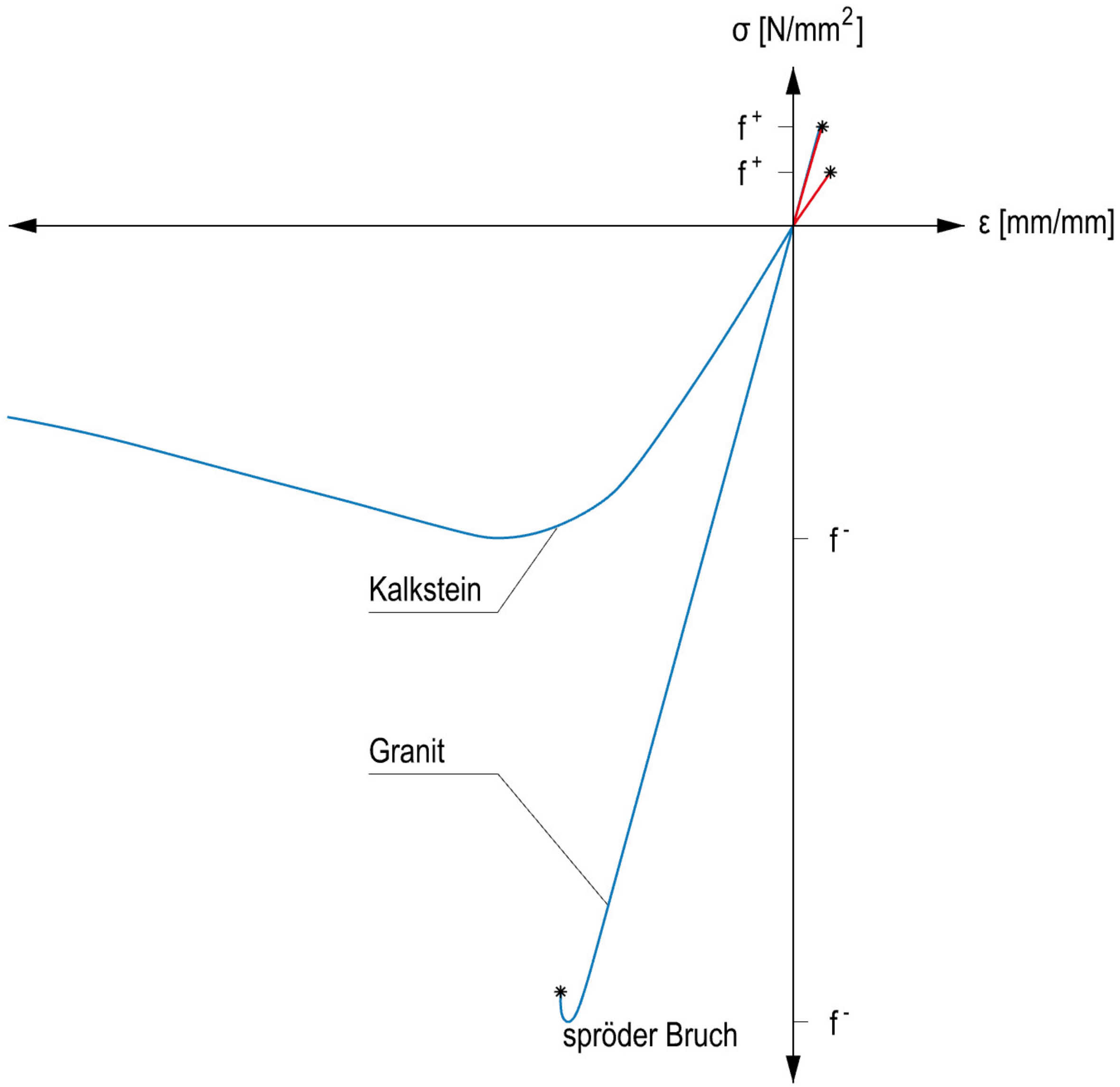
$E = 6'000 - 20'000\ N/mm^2$

Granit
Granite

$f^{+} = 2 - 15\ N/mm^2$

$f^{-} = 80 - 180\ N/mm^2$

$E = 20'000 - 50'000\ N/mm^2$



f^{+} Zugfestigkeit
Tensile strength
f^{-} Druckfestigkeit
Compressive strength
E Elastizitätsmodul
Modulus of elasticity

Materialverhalten von Stein
Material behaviour of stone

Tanne
Fir

$f^+ = 40\text{-}100\ N/mm^2$

$f^- = 20\text{-}30\ N/mm^2$

$E = 12'000\text{-}14'000\ N/mm^2$

Buche
Beech

$f^+ = 130\ N/mm^2$

$f^- = 50\ N/mm^2$

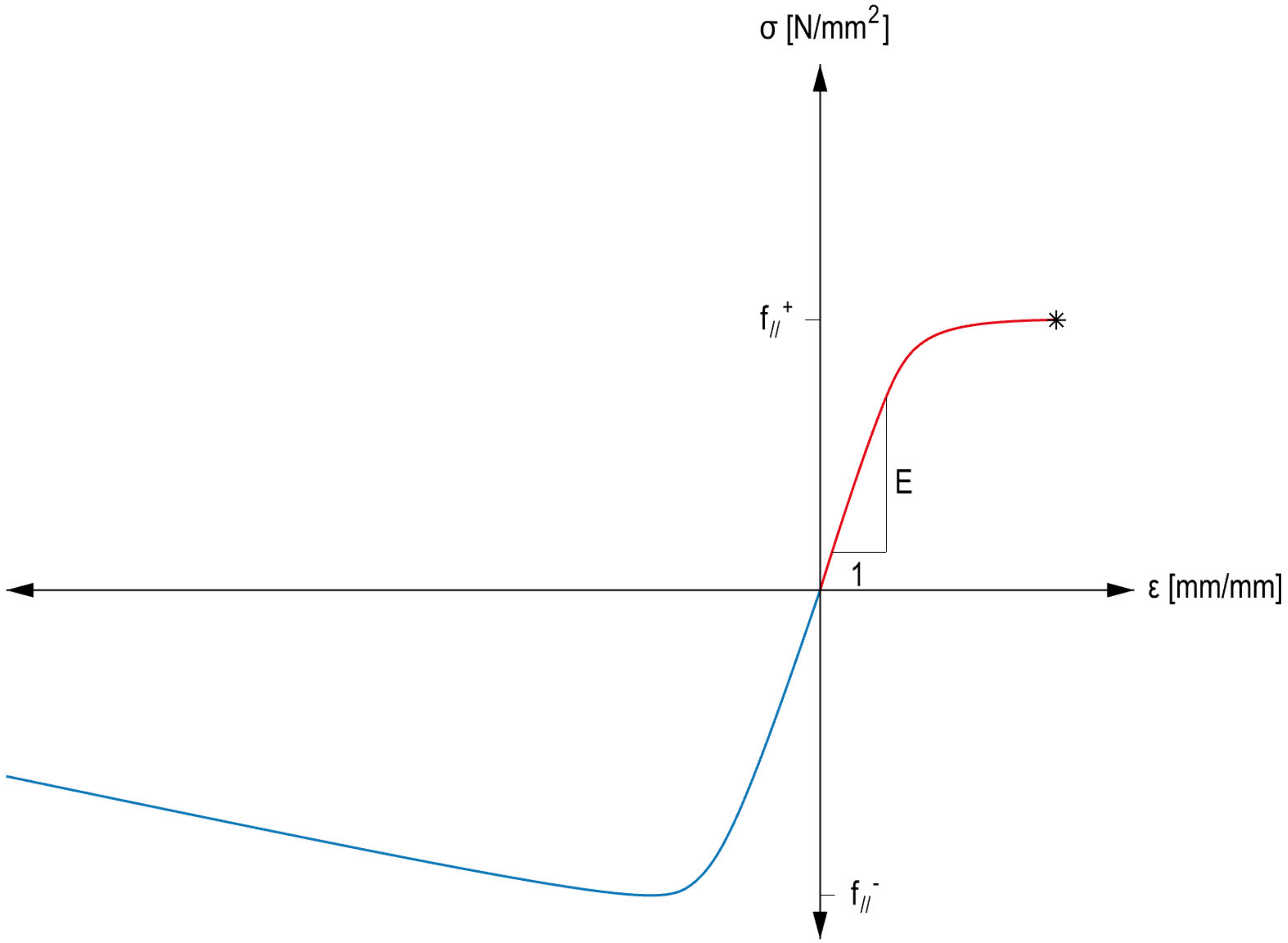
$E = 15'000\ N/mm^2$

Eiche
Oak

$f^+ = 140\ N/mm^2$

$f^- = 50\ N/mm^2$

$E = 16'000\ N/mm^2$



f^+ Zugfestigkeit
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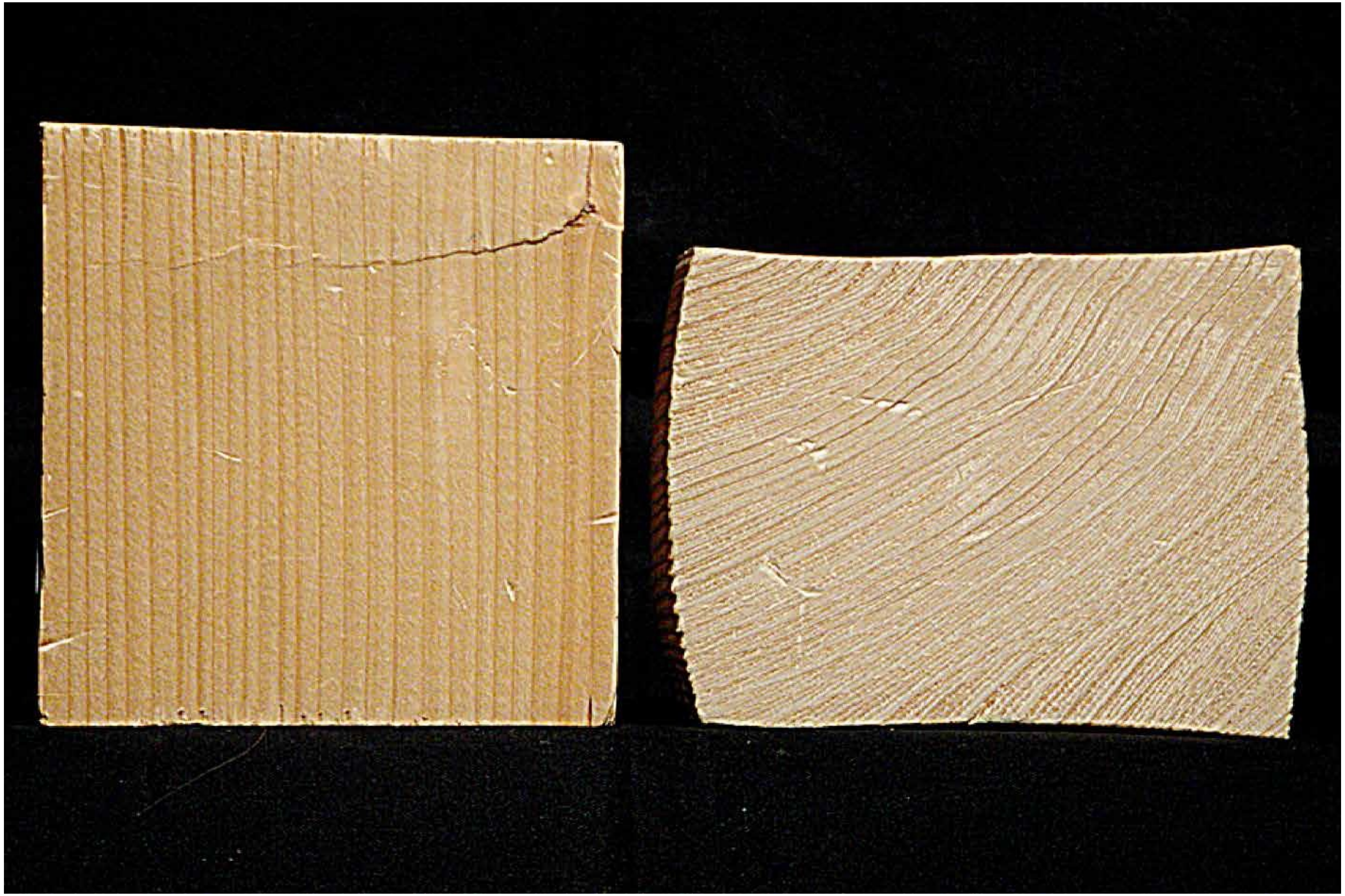
Materialverhalten von Holz
Material behaviour of wood



Materialverhalten von Holz
Material behaviour of wood

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Das Bemessungskonzept,

Beanspruchung S_d < Widerstand R_d

... gilt sowohl für den Grenzzustand der Tragsicherheit, um ein Versagen des Tragwerks zu verhindern, als auch

... für den Grenzzustand der Gebrauchstauglichkeit, um die Funktionstauglichkeit des Tragwerks sicher zu stellen.

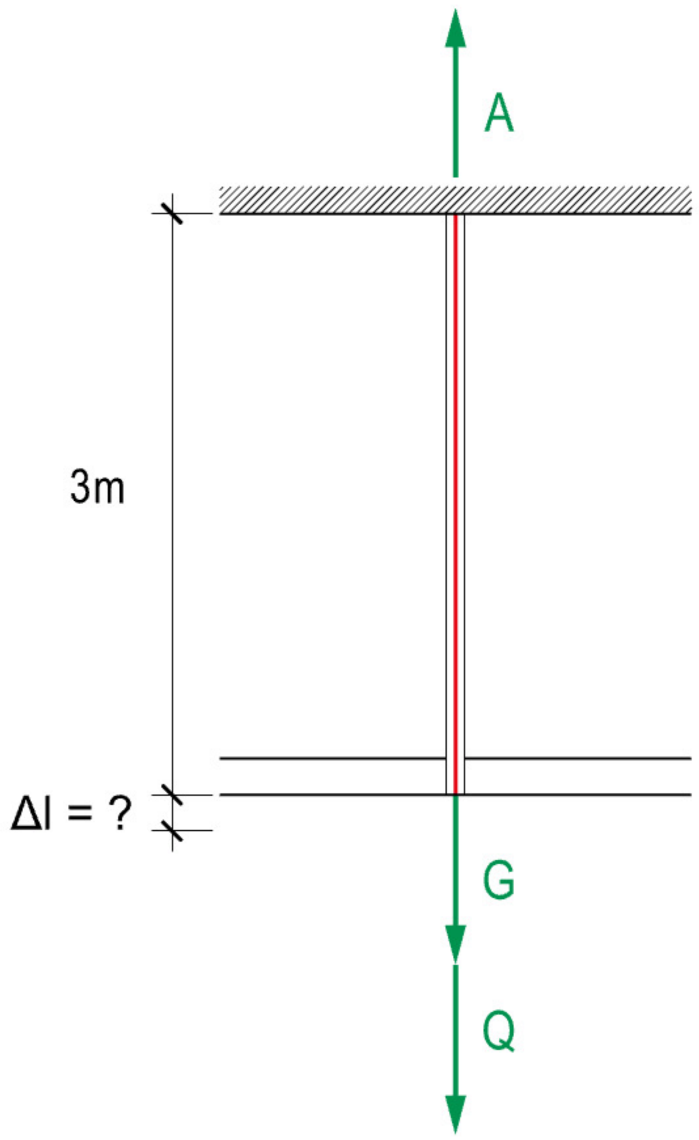
The design concept,

stress S_d < resistance R_d

... applies to both, the ultimate limit state to prevent failure of the structure, and

... for the serviceability limit state, to ensure the functional capability of the structure.

Tacoma-Narrows-Brücke, Washington, USA, 1940



Ziel der Bemessung ist
The goal of dimensioning is

- den Bruch zu vermeiden (Tragsicherheit)
  to prevent material failure (Ultimate Limit State)

- die Verformung zu begrenzen (Gebrauchstauglichkeit)
  to limit the deformation (Serviceability Limit State)

Um die Sicherheitsfaktoren erhöhte Lasten = Lasten auf Bemessungsniveau
Loads increased by the safety factors = loads at design level

$$N_d = \gamma_G * G + \gamma_Q * Q$$

Sicherheitsfaktoren
Safety factors

Ständige Lasten *Dead loads*	$\gamma_G = 1.35$
Veränderliche Lasten *Live loads*	$\gamma_Q = 1.5$

Sicherheitsfaktoren, mit welchen die Lasten zu <u>erhöhen</u> sind (CH-Norm)
Safety factors with which the loads are to be <u>increased</u> (CH standard)

Grenzzustand der Tragsicherheit
Ultimate limit state (ULS)

Tragwiderstand ermittelt mit um γ_M reduzierter Festigkeit = Tragwiderstand auf Bemessungsniveau
Load-bearing resistance determined with strength reduced by γ_M = load-bearing resistance at design level

$$N_{Rd} = f_d * A \quad \text{mit} \quad f_d = f / \gamma_M$$

Widerstandsbeiwerte
Material safety factor

Stahl *Steel*	$\gamma_M = 1.05$
Stahlbeton *Reinforced concrete*	$\gamma_M = 1.15$
Beton *Concrete*	$\gamma_M = 1.5$
Holz *Wood*	$\gamma_M = 1.5 - 1.7$
Mauerwerk *Masonry*	$\gamma_M = 2.0$

Sicherheitsfaktoren, mit welchen die Materialfestigkeiten zu reduzieren sind (CH-Norm)
Safety factors with which the material strengths are to be reduced (CH standard)

$$N_d = \gamma_G * G + \gamma_Q * Q$$

$$N_{Rd} = f_d * A \quad \text{mit} \quad f_d = f / \gamma_M$$

Sicherheitsfaktoren
Safety factors

Ständige Lasten *Dead loads*	$\gamma_G = 1.35$
Veränderliche Lasten *Live loads*	$\gamma_Q = 1.5$

Widerstandsbeiwerte
Material safety factor

Stahl *Steel*	$\gamma_M = 1.05$
Stahlbeton *Reinforced concrete*	$\gamma_M = 1.15$
Beton *Concrete*	$\gamma_M = 1.5$
Holz *Wood*	$\gamma_M = 1.5 - 1.7$
Mauerwerk *Masonry*	$\gamma_M = 2.0$

Grenzzustand der Tragsicherheit
Ultimate limit state (ULS)

Die Bemessungs-Beanspruchung muss in jedem Fall kleiner sein als der Tragwiderstand ($N_{Rd} \geq N_d$).
The design load must in any case be smaller than the load-bearing resistance ($N_{Rd} \geq N_d$).

Die erforderliche Querschnittsfläche eines Tragelementes kann somit sehr einfach durch Division der Bemessungs-Beanspruchung durch die Bemessungs-Festigkeit f_d ermittelt werden:
The required cross-sectional area of a load-bearing element can thus be determined very simply by dividing the design load by the design strength f_d:

$$A_{erf} \geq \frac{N_d}{f_d}$$

Diese Bemessungs-Bedingung wird als Kriterium der Tragsicherheit bezeichnet.
This design condition is referred to as the ultimate limit state criterion.

Lasten auf Gebrauchsniveau
Loads at serviceability state

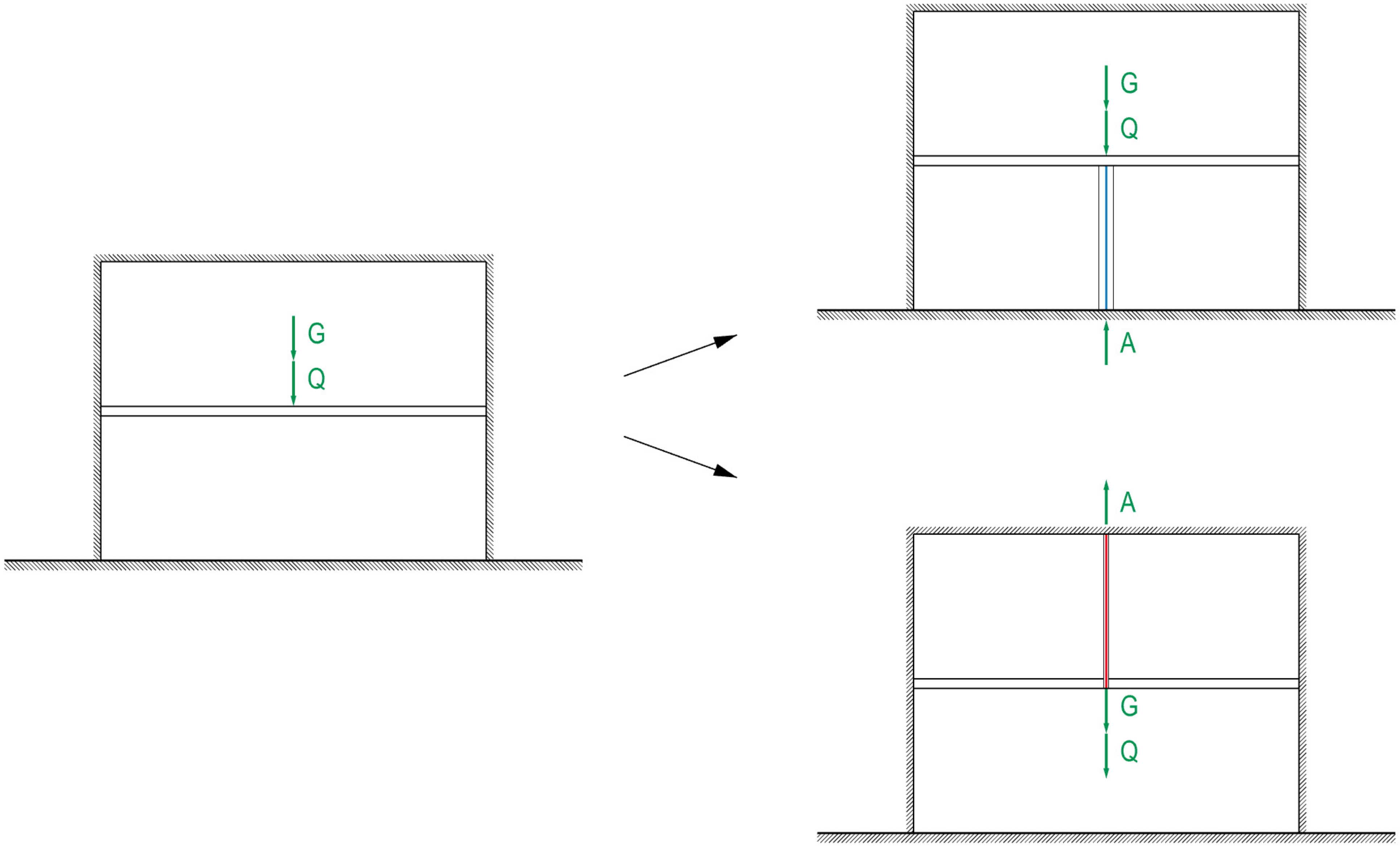
$$N = G + Q$$

Bedingung für den Zustand der Gebrauchstauglichkeit:
Condition for the serviceability state:

Die Verformungen unter Gebrauchslasten müssen kleiner sein als die zulässigen Verformungen.
The deformations under live loads must be smaller than the allowed deformations.

$$\Delta l \leq \Delta l_{zul}$$

Diese Bemessungs-Bedingung wird als Kriterium der Gebrauchstauglichkeit bezeichnet.
This design condition is referred to as the serviceability limit state criterion.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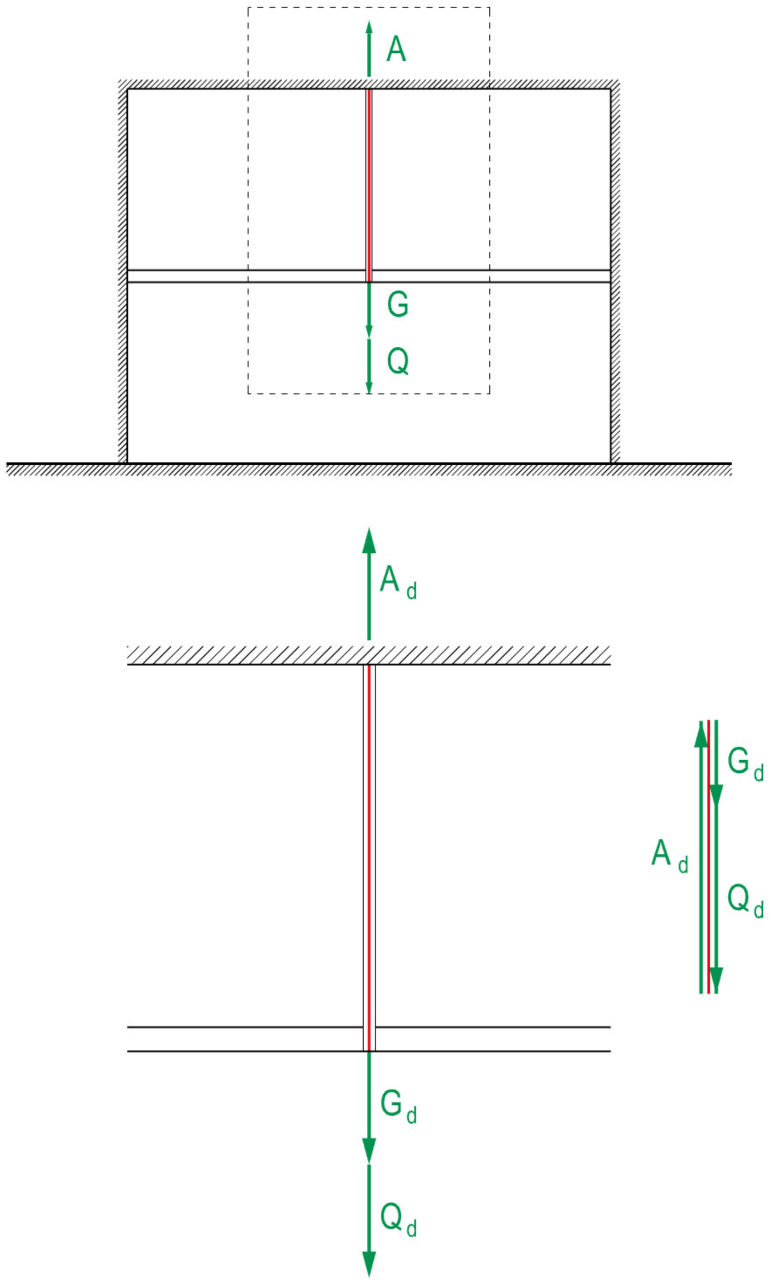
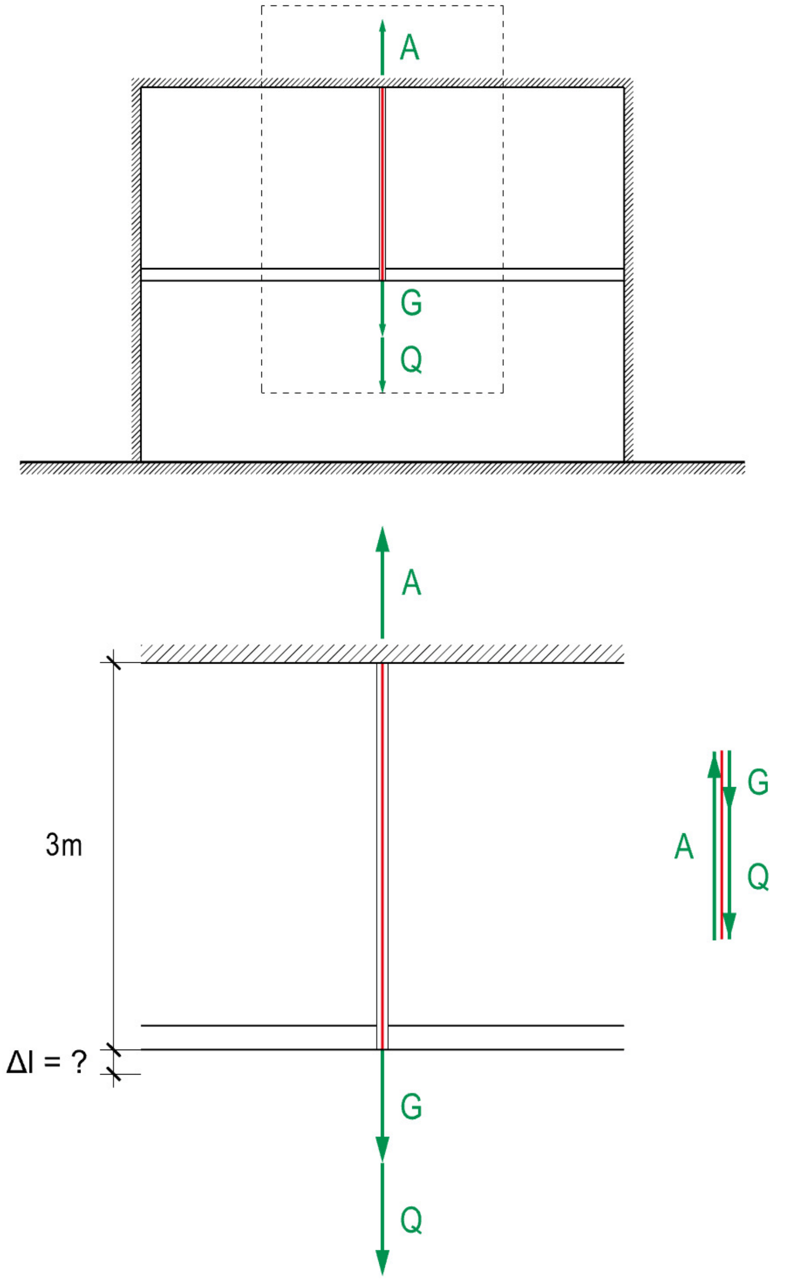
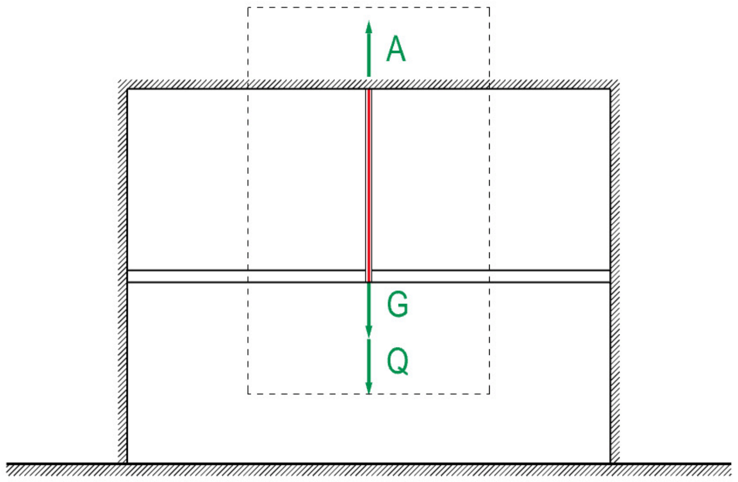


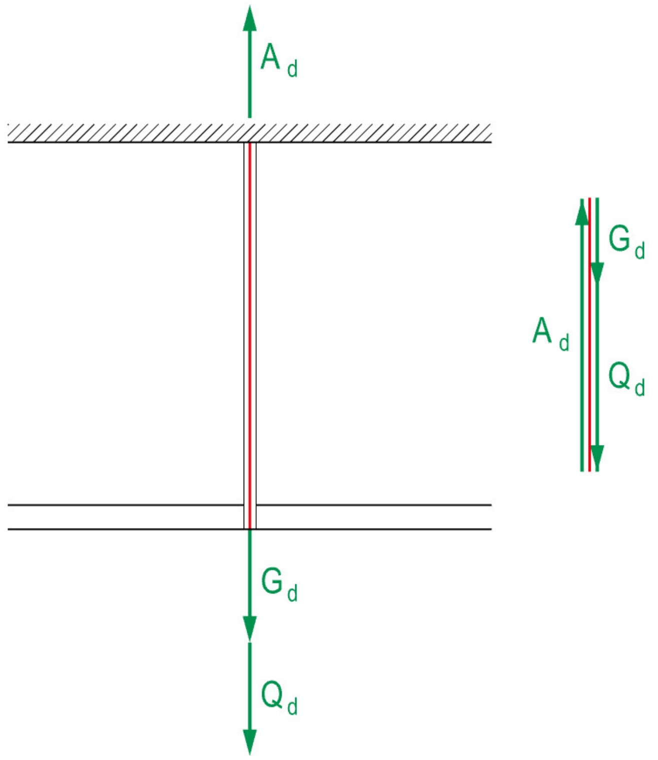
Grenzzustand der Tragsicherheit

Ultimate limit state (ULS)



Grenzzustand der Gebrauchstauglichkeit
Serviceability limit state (SLS)





$G = 25\ kN = 25‘000\ N$

$Q = 50kN = 50‘000\ N$

$$\boxed{N_d = G * \gamma_G + Q * \gamma_Q}$$

$N_d = 25‘000\ N * 1.35 + 50‘000\ N * 1.5 = 108‘750\ N$

$f\ = 355\ N/mm^2$

$\gamma_M = 1.05$

$$\boxed{f_d = f/\ \gamma_M}$$

$$f_d = \frac{355\ N/mm^2}{1.05} = 338\ N/mm^2$$

$$\boxed{A_{erf} \geq\ N_d/f_d}$$

$$A_{erf} \geq \frac{108‘750\ N}{338\ N/mm^2} = 322\ mm^2$$

$322\ mm^2 = \pi * r^2$

$r \geq 10\ mm;\ D \geq 20\ mm$

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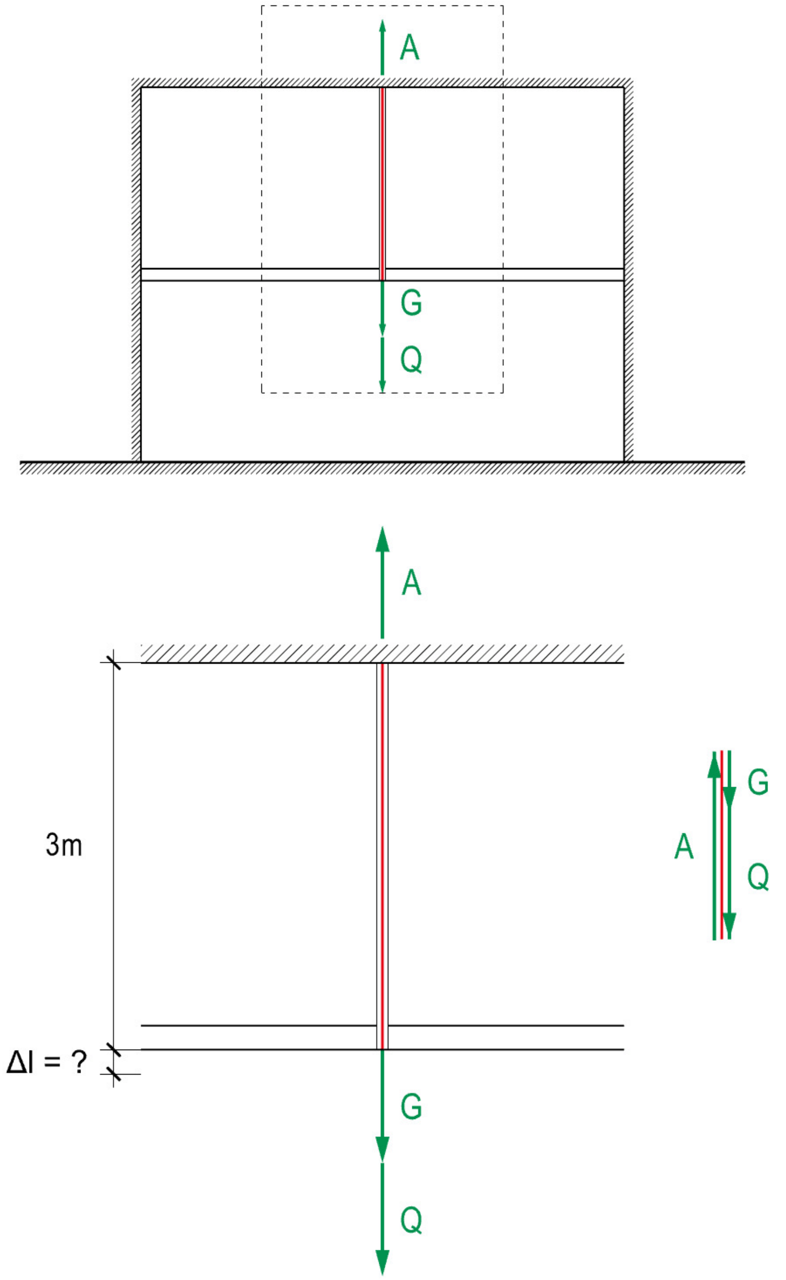


$G = 25\ kN = 25'000\ N$
$Q = 50kN = 50'000\ N$

$$A_{erf} \geq \frac{N}{E} * \frac{l}{\Delta l}$$

$\Delta l_{\ zul} = 10\ mm$

$$A_{erf} \geq \frac{(25'000\ N + 50'000\ N)}{210'000\ N/mm^2} * \frac{3'000\ mm}{10\ mm} = 107\ mm^2$$

$A_{erf} \geq 107\ mm^2$

$107\ mm^2 = \pi * r^2$
$r \geq 6\ mm;\ D \geq 12\ mm$

SLS: $D \geq 12\ mm$

ULS: $D \geq 20\ mm$

Grenzzustand der Gebrauchstauglichkeit
Serviceability limit state (SLS)

Material und Bemessung

Material and dimensioning

Innere Kräfte und Verformung im Tragelement
Internal forces and deformation in a structural element

Steifigkeit des Tragelements
Stiffness of a structural element

Innere Kräfte im Material
Internal forces in the material

Verformung und Steifigkeit des Materials
Deformation and stiffness of the material

Materialverhalten verschiedener Baustoffe
Behaviour of different building materials

Bemessung für Grenzzustände
Dimensioning for limit states

>> Verformung aufgrund Temperaturveränderung
Deformation caused by temperature changes

Ermüdung
Fatigue

Holz *Timber*	$\alpha_T = 0.0005\ \%/C^0$
Glas *Glass*	$\alpha_T = 0.0007\ \%/C^0$
Stein (Granit) *Stone (Granite)*	$\alpha_T = 0.0008\ \%/C^0$
Beton *Concrete*	$\alpha_T = 0.0010\ \%/C^0$
Stahl *Steel*	$\alpha_T = 0.0010\ \%/C^0$
Aluminium *Aluminium*	$\alpha_T = 0.0024\ \%/C^0$

Wärmeausdehnungskoeffizient
Temperature expansion coefficient

$$\varepsilon = \frac{\Delta l}{l} = \alpha_T * \Delta T$$

Für Stahl und Beton gilt:
The following applies to steel and concrete:

$$\alpha_T = 0.00001 \ [°C^{-1}] = 0.001 \ [\% \ / \ °C] = 0.01 \ [‰ \ / \ °C]$$

$$\varepsilon_T = \frac{\Delta l}{l} = \alpha_T * \Delta T$$

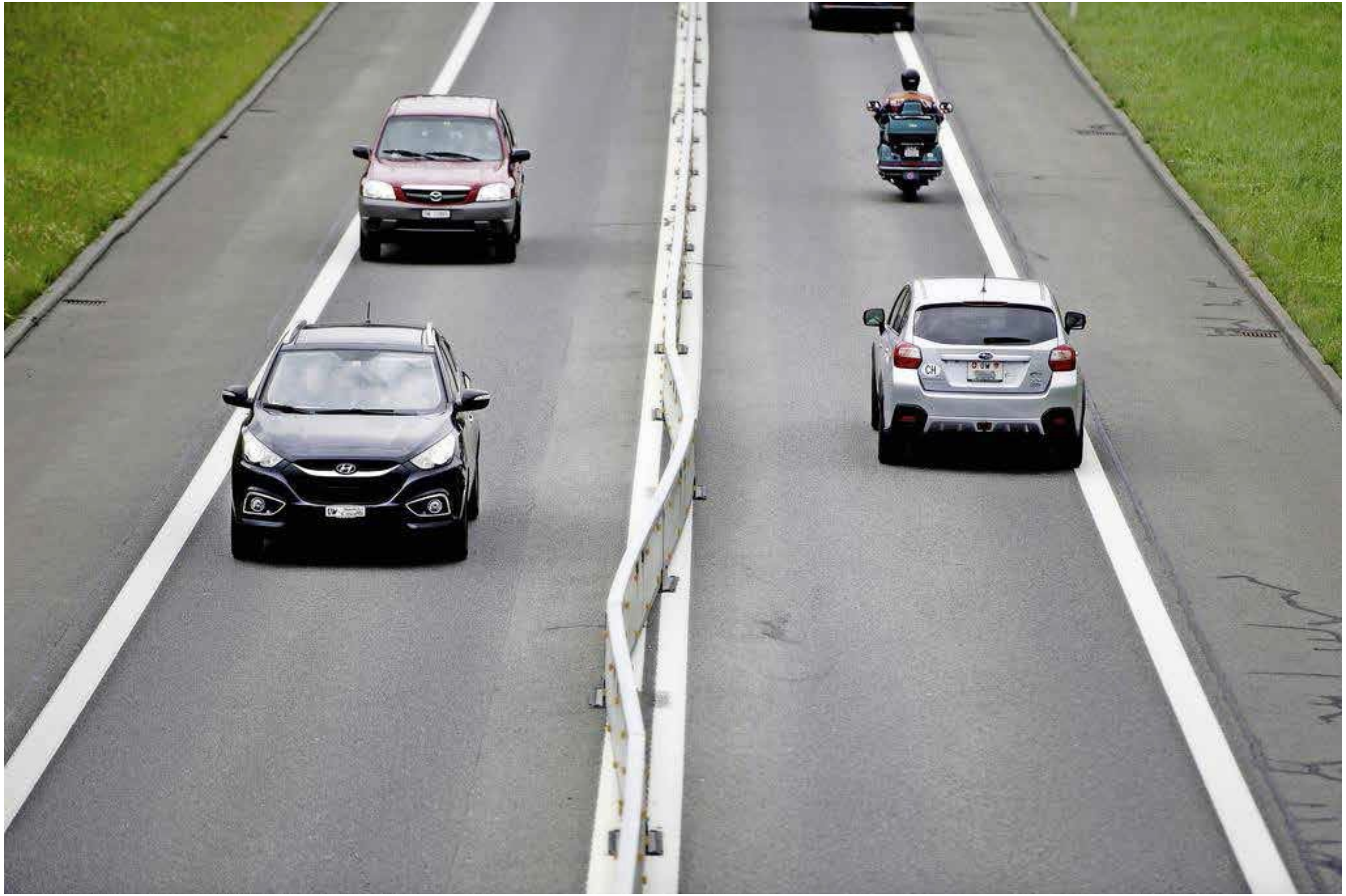
Beispiel für die Wärmeausdehnung einer Stahlstrebe
Example of the thermal expansion of a steel strut

$l = 5$ m
$\alpha_T = 0.00001$ [°C^{-1}]
$\Delta T = 70°$ (the steel strut is heated from -20°C to +50°C)

$\Delta l = \alpha_T * \Delta T * l = 0.00001$ [°C^{-1}] $* 70$ [°C] $* 5$ [m] = <u>3.5 mm</u>

Verformung aufgrund Temperaturveränderung

Deformation caused by changes in temperature

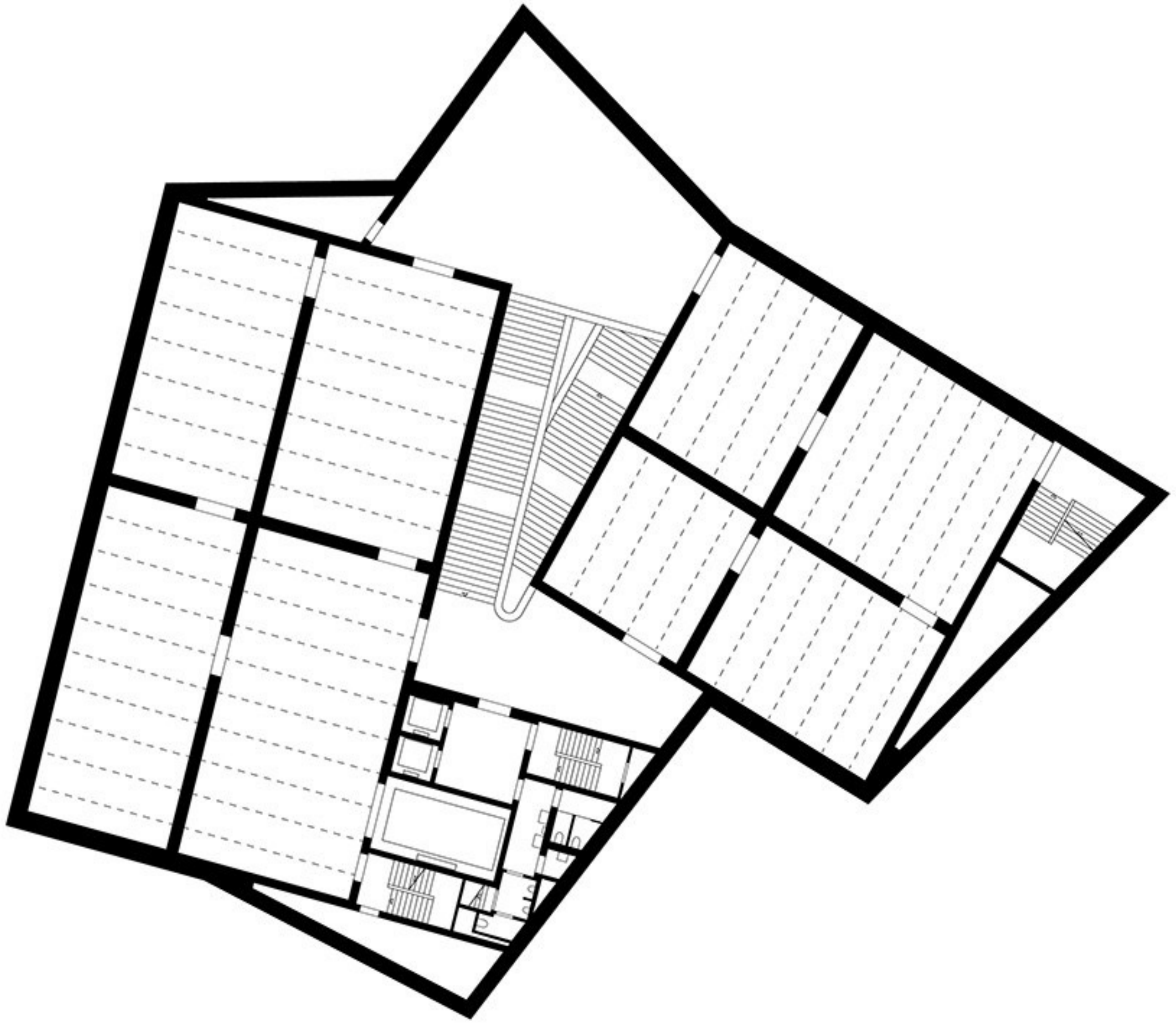


Dilatationsfuge, Louis Kahn, Ayub National Hospital, Dhaka, 1965-1969

Dilatationsfuge

Expansion joint

Erweiterung Kunstmuseum Basel, Christ & Gantenbein, ZPF Ingenieure



Erweiterung Kunstmuseum Basel, Christ & Gantenbein, ZPF Ingenieure

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