Proceedings of the IASS-SLTE 2014 Symposium
"Shells, Membranes and Spatial Structures: Footprints"
15 to 19 September 2014, Brasilia, Brazil
Reyolando M.L.R.F. BRASIL and Ruy M.O. PAULETTI (eds.)

Compression-only Form finding through Finite Subdivision of the Force Polygon

Masoud AKBARZADEH*, Tom VAN MELE [a], Philippe BLOCK [a],

* ETH Zurich, Institute of Technology in Architecture, BLOCK Research Group
Stefano-Franscini-Platz 5, HIL H 46.2, CH-8093 Zurich, Switzerland
amasoud@ethz.ch

[a] ETH Zurich, Institute of Technology in Architecture, BLOCK Research Group

Abstract

This paper demonstrates the potential of combining graphic statics with subdivision schemes to generate exciting compressive solutions for given loading and boundary conditions in two dimensions. It emphasizes the independence of the internal and external equilibrium polygons in the force diagram of compression-only structures and suggests a design strategy by reconfiguring the internal force polygons through subdividing the space bounded by the external force polygon. The resulting structural forms demonstrate the strength of graphical methods for design and form finding of structures.

Keywords: Internal force polygons, finite subdivision, compression-only structures, external force polygon, graphical methods.

1. Introduction

Graphical methods allow the structural design of expressively elegant and structurally efficient forms. Graphic statics is a collection of geometric recipes established in 19th century (Culmann [6]; Cremona [5]; Wolfe [14]). It uses two geometrically dependent/reciprocal diagrams: the form diagram represents the geometry of a structure, and the force diagram visualizes the equilibrium of internal and external forces (Maxwell [8]). The geometric dependencies between the form and the force diagrams allow designers to manipulate these diagrams and visually observe the effects of this manipulation as a change in (the) geometry (of the structural system) or the magnitude of the forces (Van Mele *et al.* [12]). For this reason, graphic statics is considered an intuitive method for structural design among engineers and architects.

For given boundary conditions (such as the magnitude, number and location of applied forces and supports), 2D/3D graphic statics can either be used to analyze an existing form or to design an efficient structural form. The latter approach uses the force diagram to derive the geometry of the form and thus has the following advantages.

- The force diagram includes information about the magnitude of internal and external forces, which can be creatively used in the design of efficient structures. For instance, the Michell truss and the constant–force truss are renowned examples in which the magnitudes of forces are geometrically controlled in the force diagram from which the structure's geometry then follows (Michell [9]; Allen and Zalewski [2]).
- Manipulating the force diagram can result in unique design features in the derived form. For instance, Block and Ochsendorf [4] and Rippmann *et al.* [10] showed that attracting a group of forces in the force diagram can result in generating creases in the geometry of free–form shell structures.
- Building up the force diagram of closed polygons guarantees the equilibrium of forces in the derived form. For instance, the aggregation of convex force diagrams (polyhedrons) can generate spatial compression/tension-only forms (Akbarzadeh *et al.* [1]).

Although using the force polygon in form finding is a common technique among architects and engineers, less attention has been given to the layers of hidden information embedded in the force diagram. Therefore, the main objective of this paper is to investigate and emphasize the properties of the force diagram and use these to develop a design approach for deriving non-conventional structural forms.

This research considers the force diagram as a design apparatus and introduces an inventive additional step in the conventional process of graphic statics. It addresses global and local equilibrium of forces separately in the force diagram and uses this property to design intricate, compression-only structural forms. To emphasize the applicability and reproducibility of the approach introduced in this paper, all presented drawings have been constructed using common drafting techniques in the parametric environment of GeoGebra 4.4 (Hohenwarter *et al.* [7]).

The outline of this paper is as follows. In Section 2.1, the characteristics of the force polygon and its external and internal elements are briefly described. In Section 2.2, the idea of subdividing the force polygon as a method of design for generating compression/tension-only structural forms is introduced. In Section 3, the design applications of the discussed method are presented for three cases of boundary conditions for each of which various structural forms are generated.

2. Methodology

2.1. Internal and external equilibrium

According to the reciprocal relationship between form and force diagrams, any closed polygon in the force diagram represents the equilibrium of the corresponding group of forces in the form diagram. Therefore, it is possible to distinguish between the internal and external forces by their force polygons in the force diagram. Moreover, for given boundary conditions, the equilibrium of the applied (external) forces is independent from the form of the structure; i.e., there are infinite structural forms that can be in equilibrium for those given boundary conditions. Reciprocally, for the force diagram, the configuration of the external force polygon is independent from the configuration of the internal force polygons.

As an example, Figure 1a shows the form (left) and the force (right) diagrams of a funicular arch for a given loading condition. The highlighted (external) polygon in the force diagram consists of all peripheral edges, and represents the equilibrium of the applied loads and reaction forces. The force diagrams in Figure 1a–c have the same external polygon. However, they have different configurations of internal polygons, and, therefore, different structural forms for the same boundary conditions. The internal configuration of the force diagram corresponds to the form of the structure, and the external polygon of the force diagram is reciprocal to the boundary conditions of the form diagram. In this regard, designing the internal configuration of the force diagram can be considered a promising approach in deriving various structural forms.

2.2. Subdividing the interior space of the external force polygon

In compression or tension-only structures, the force diagram consists of convex polygons (Whiteley et al. [13]; Van Mele and Block [11]). Note that also the form diagram has to consist of convex and unbounded polygons.

Changing the form of the structure by changing the configuration of the internal space of a force polygon is allowed, only if it induces a new state of equilibrium within the force diagram. For example, in order to find a compression-only form for given boundary conditions, the internal space of the external force polygon should be designed such that it consists of only closed convex polygons.

In the force diagram, any subdivision of the internal space, bounded by the peripheral edges, represents internal equilibrium and determines the form of the structure. For instance, Figure 1b represents the form and the force diagrams that resulted from barycentric subdivision of the force diagram of Figure 1a. Figure 1c contains the form and the force diagrams that resulted from barycentric subdivision of the force diagram of Figure 1 b.

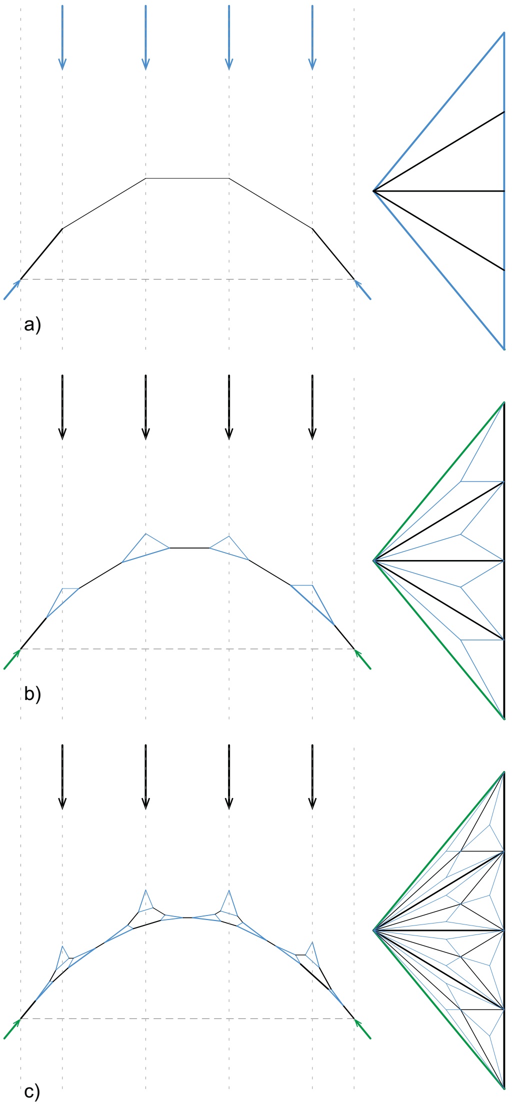


Figure 1: a) Form and force diagrams of a funicular arch under applied vertical loads; b) the form and the force diagram after barycentric subdivision of the force polygon in (a); and c) the form and the force diagrams after barycentric subdivision of the force polygon in (b).

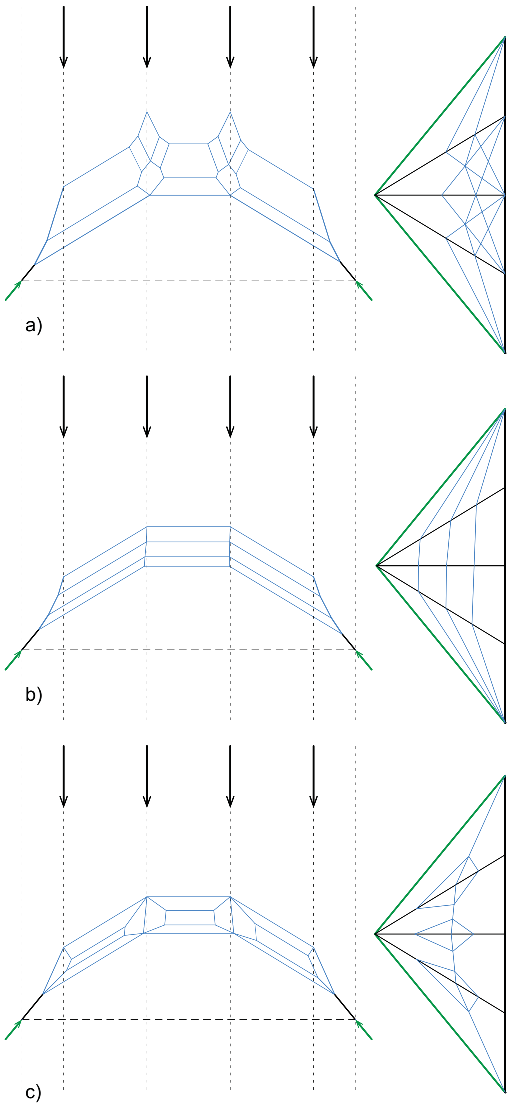


Figure 2: Form and force diagrams resulting from different subdivision rules: a) connecting the mid points of the internal edges to the end points of the external edges; b) connecting the midpoints of the adjacent internal edges and eventually the end points of the external edges; and c) adding closed polygons around the vertices generated in the first level of subdivision of the force polygon of (b).

Although many subdivision algorithms exist in mathematics, which could be used to subdivide the internal space of a polygon, the following rules should be followed to ensure the state of equilibrium:

- The edges of the external force polygon cannot be divided; dividing these would change the number and magnitude of the loads and/or the reaction forces and therefore does not preserve the given boundary conditions.
- Only internal subdivisions that generate convex polygons are permitted. Each internal angle of the generated polygon should be less than 180 degrees.

In the next section, some simple subdivision algorithms are used to divide the force polygon and generate design examples for compression/tension-only structural forms.

3. Design Applications

Subdividing the internal space of the external force polygon provides an opportunity to explore a variety of structural forms without changing the boundary conditions for a given problem. In the following sections, various structural designs are demonstrated by subdividing the force polygon for different boundary conditions.

3.1. Arching structures

Figure 2a–c represent form and force diagrams resulting from different types of subdivision techniques applied to the force diagram of the simple funicular arch in Figure 1a. In Figure 2a, the form diagram results from connecting the midpoints of the internal edges to the end points of the external edges of the force polygon. Connecting points to each other and the end points of the external force diagram generates the compression-only form of Figure 2b. Adding closed polygons to the subdivision rule in Figure 2b produces the form diagram of Figure 2c.

3.2. Branching structures

The same technique can be used to generate sophisticated, branching structural forms. Figure 3a represents the form and force diagrams of a compression-only branching structure. Usually, in these types of structures, the top chord is in tension, and the branching body is in compression. In such cases, it is possible to pre-stress the top chord with a tension tie (blue forces), making the whole structure act in compression. Using graphic statics, the magnitude of this pre-stressing force can be controlled by the designer as a degree of freedom in design. Barycentric subdivision of the force polygons in this example generates a branching system as illustrated in Figure 3b. Combining the subdivision rules in an additive and recursive manner generates more complex branching systems as shown in Figures 4 and 5 respectively.

3.3. Fan-like structures

For the same boundary conditions, fan-like structural forms can also be generated. Note that the reciprocity between the form and the force diagrams allows constructing a force diagram that contains an arch and corresponds to a fan-like structural form (Figure 6). This is very similar to the railway bridge designed by I. K. Brunel (Allen and Zalewski [2]). Further subdivision of the force diagram by constructing shallower funicular arches results in the form diagram of Figure 6b. Using various techniques to subdivide the force diagram of Figure 6a can result in more surprising structural forms (Figure 7).

3.4. Combined arching and branching structures

It is possible to design structural forms that can be visually considered as the combination of all previous examples. Figure 8a–c represent the form and force diagrams for the following conditions: three support forces (green), two pre-stressing forces on the top chord (blue), and vertical applied forces (black). The magnitude of the horizontal reaction force on the top corner of the structure can be considered as an extra degree of freedom in the force polygon. Further subdivision of the force polygon can result in various structural forms that combine the idea of branching systems with funicular arches.

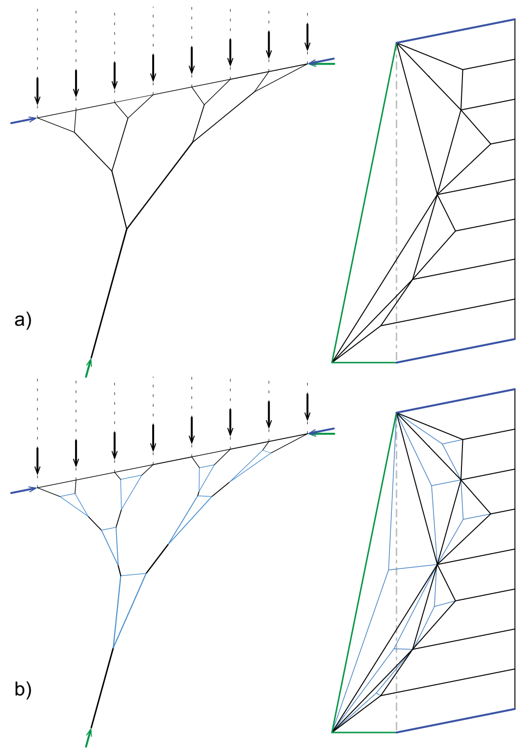


Figure 3: a) The form and the force diagrams of a branching structure with two additional pre-stressing forces (blue) at the top chord; and b) the form and the force diagrams resulting from a barycentric subdivision of the force polygon of (a).

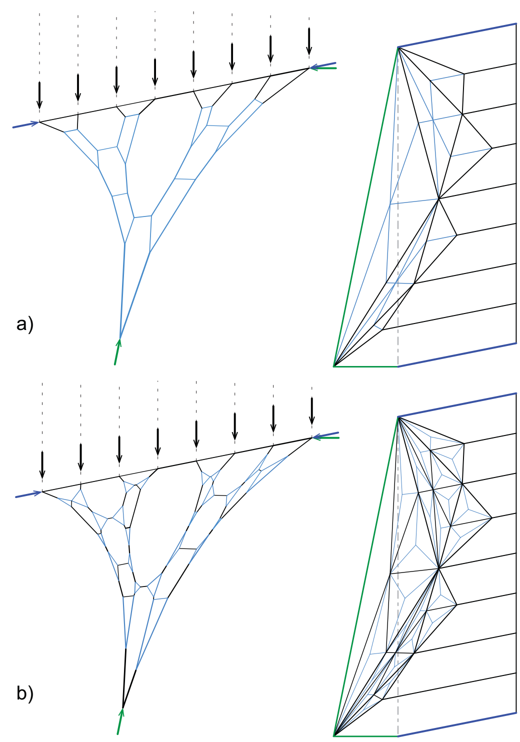
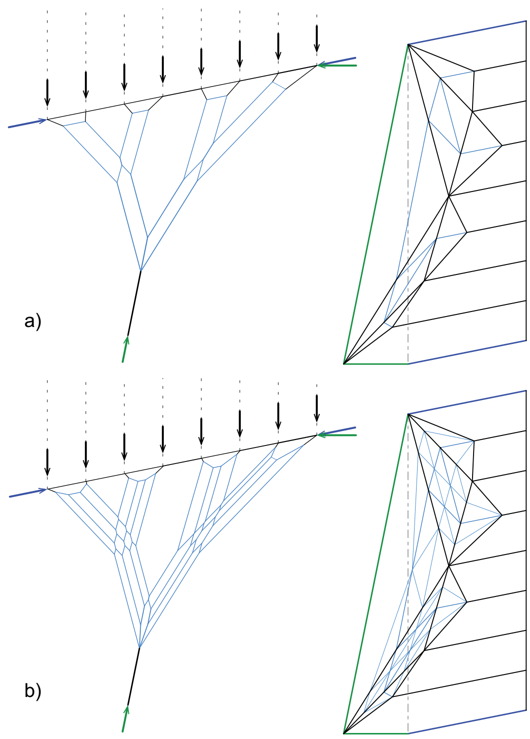


Figure 4: a) The form and the force diagrams resulting from connecting centroids, midpoints, and apexes of each internal polygon; and b) the form and the force diagrams resulting from barycentric subdivision of the force polygons generated in (a).



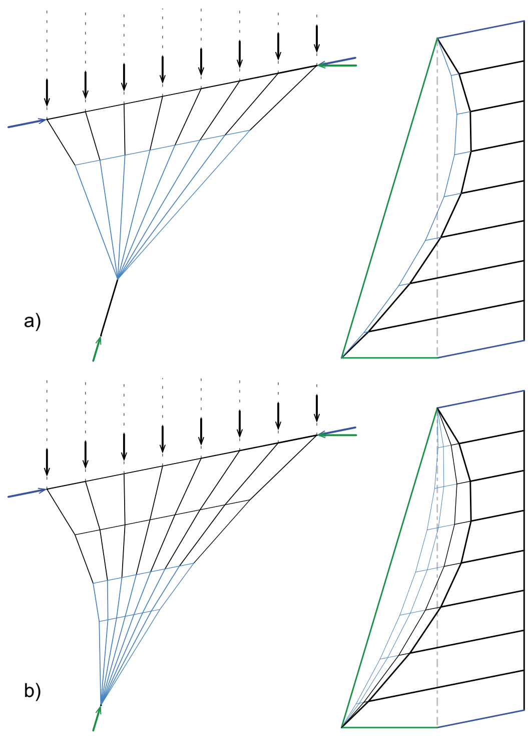


Figure 5 (previous page): a) The form and the force diagrams resulting from connecting the mid points of the edges of each internal triangle; and b) the form and the force diagrams resulting from recursively subdividing the polygons generated in (a).

Figure 6 (previous page): a) The form and the force diagrams for fan-shaped compression forms; and b) the form and the force diagrams resulting from recursively subdividing the force polygons generated in (a).

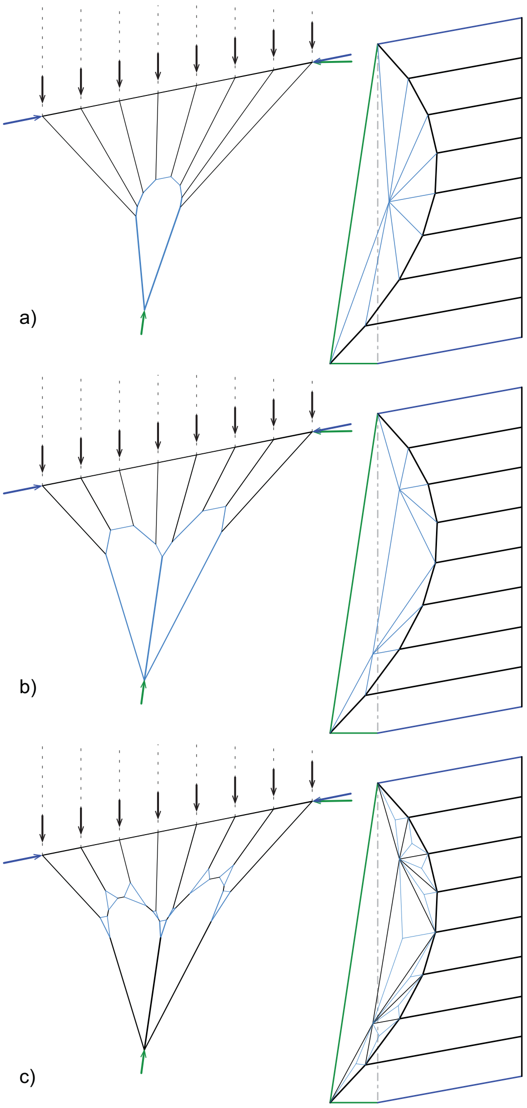


Figure 7: a) The form and the force diagrams resulting from barycentric subdivision of the force polygon in Figure 6 a; b) different subdivision technique applied to the same figure; and c) combining the subdivision techniques used in (a) and (b).

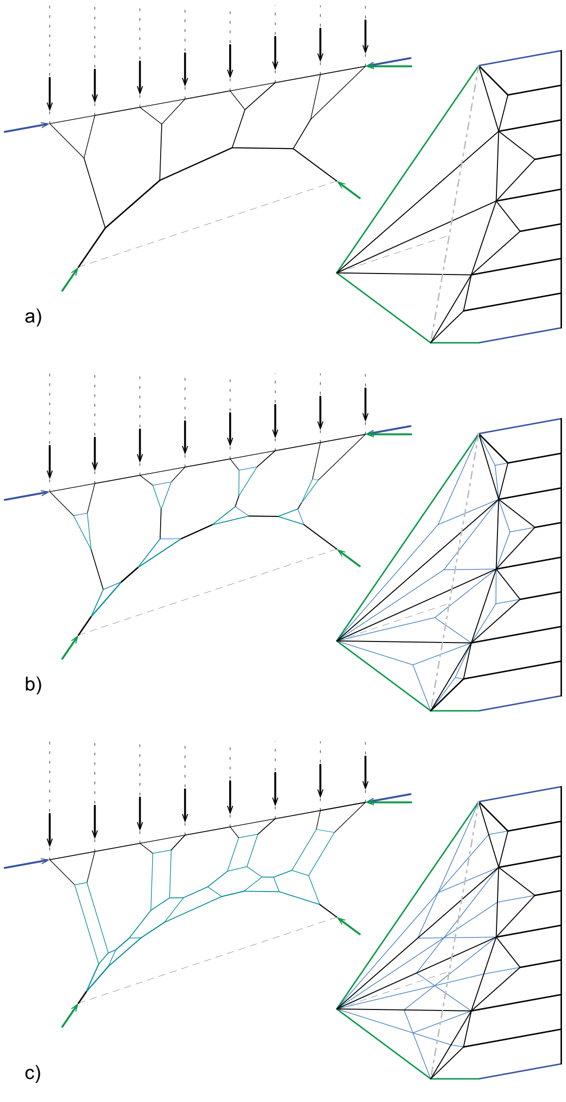


Figure 8: a) The form and the force diagrams for boundary conditions with three supports, a tension tie, and vertical applied forces; b) and c) the form and the force diagrams resulting from various subdivision algorithms applied to the force diagram of (a).

4. Conclusion and Discussion

This research showed that subdivision of the interior space of the external force polygon can be used as a strategy for the design of compression/tension-only structural forms. It, thereby, emphasizes the process of reconfiguring the internal force polygons. As a result, it provides a range of various novel structural forms for the three different cases of boundary conditions.

It, therefore, provides a design strategy that not only can be used in the common manual process of graphic statics, but also be implemented computationally for more sophisticated designs. Moreover, these structural forms can be optimized further by minimizing the load path of the structure as suggested by Beghini *et al.* [3], e.g. by writing the dependencies between the form and force diagrams algebraically (Van Mele *et al.* [11]).

References

[1] Akbarzadeh M., Van Mele T. and Block P., Equilibrium of spatial structures using 3-D reciprocal diagrams, in *Proceedings of IASS Symposium 2013, BEYOND THE LIMITS OF MAN*, Obrebski J.B. and Tarczewski R. (eds.), Wroclaw University of Technology, Poland, 2013.

[2] Allen E. and Zalewski W., *Form and Forces: Designing Efficient, Expressive Structures*. John Wiley Sons, New York, 2010.

[3] Beghini L. L., Carrion J., Beghini A., Mazurek A. and Baker W. F., Structural optimization using graphic statics. *Structural and Multidisciplinary Optimization*, 2013.

[4] Block P. and Ochsendorf J., Thrust Network Analysis: A new methodology for three-dimensional equilibrium. *Journal of the International Association for Shell and Spatial Structures*, 2007; **48**(3); 167-173.

[5] Cremona L., *Graphical Statics: Two Treatises on the Graphical Calculus and Reciprocal Figures in Graphical Statics*. Clarendon Press, Oxford, 1890.

[6] Culmann K., *Die Graphische Statik*. Verlag Meyer & Zeller, Zurich, 1864.

[7] Hohenwarter M., Borcherds M., Ancsin G., Bencze B., Blossier M., Delobelle A., Denizet C., Éliás J., Fekete Á. , Gál L., Konečný Z., Kovács Z., Lizelfelner S., Parisse B. and Sturr G., GeoGebra 4.4, December 2013; http://www.geogebra.org.

[8] Maxwell J. C., On reciprocal figures and diagrams of forces. *Philosophical Magazine and Journal Series*, 1864; **4**; 250-261.

[9] Michell A. G. M., The limits of the economy of the material. *Philosophical Magazine*, 1904; **8**; 589-597.

[10] Rippmann M., Lachauer L. and Block P., Interactive vault design. *International Journal of Space Structures*, 2012; **27**; 219-230.

[11] Van Mele T. and Block P., Algebraic graph statics. *Computer-Aided Design*, 2014; **53**; 104-116.

[12] Van Mele T., Rippmann M., Lachauer L. and Block P., Geometry-based understanding of structures. *Journal of the International Association of Shell and Spatial Structures*, 2012; **53**(4); 285-295.

[13] Whiteley W., Ash P. F., Bolker E. and Crapo H., Convex polyhedra, Dirichlet tessellations, and spider webs. In M. Senechal, editor, *Shaping Space: Exploring Polyhedra in Nature, Art, and the Geometrical Imagination*. Springer New York, 2013.

[14] Wolfe W. S., *Graphical Analysis: A Text Book on Graphic Statics*. McGraw-Hill Book Company, Inc., 1921.