



Form Finding to Fabrication:

A digital design process for masonry vaults

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Abstract

It is common practice in engineering to represent a continuous stress field by a network of discrete forces, e.g. hanging models (Gaudí), cable nets (Otto) or strut-and-tie models (Schlaich). These discrete force networks allow the user to visualize, grasp and control the flow of forces in an intuitive manner. A recent computational advance in this field is Thrust Network Analysis (TNA), which allows form finding controlled through discrete force diagrams.

In this paper, a fully digital design process –from form finding to fabrication– for compression-only masonry shells will be presented. The challenge is to provide a smooth and efficient generation of different data representations of the discrete geometry of the force (or thrust) network for the use in structural and experimental modelling, discrete element modelling (DEM) software, or fabrication. The TNA method has been implemented in existing CAD software, offering fully interactive control of the boundary conditions, the internal stress field and the weight distribution of the shell. The discrete thrust network is linked to a NURBS surface to derive continuous geometric information, hence allowing the automatic generation of the shell volume according to its self weight.

Keywords: digital workflow, form-finding, parametric modelling, masonry vault design, fabrication

1 Introduction

The direct integration of structural constraints in the design process using physical models has a long tradition. In 1908, Antoni Gaudí finished a sophisticated hanging model for the design of the Colonia Güell church [1]. In 1964, Frei Otto founded the Institute of Lightweight Structures (IL) at the University of Stuttgart, where the use of physical models in structural design has been studied systematically [2]. A first application of these model-based design processes for shell design from the IL has been the building *Multihalle Mannheim* in 1975.

In contrast to Otto, the Swiss engineer Heinz Isler used since the 1950s a model-based workflow not for lightweight structures, but for thin concrete shell design. Isler worked



not only on the form finding, but also on the in-depth optimization of a continuous design-analysis-construction process based on physical models. For the design phase, Isler used inverted hanging cloth models, hardened with plaster or polyester resin, which then were tested with electronic strain gauges to measure the shell behaviour under applied load. Finally, the geometry was carefully measured by a mechanical, three-dimensional measurement device, which traced the profiles used for the scaffolding plans [3].

Today, in the age of computer aided design and manufacturing, the concept of the continuous *digital chain* has been introduced for architectural design [4]. The integration of geometric and fabrication constraints in a digital workflow has been shown for timber construction [5]. Recently, the integration of structural constraints in a design process using a digital simulation of hanging models and linking them to fabrication has been demonstrated [6].

The use of simulations and computational tools, instead of physical models and mechanical tools, causes a shift in the problem. While Isler mainly had to overcome inaccuracies in the modelling process and the material testing, the challenges today are in data processing, as each part of the process (e.g. form finding, structural analysis, and manufacturing) uses different data representations, and each conversion of data is typically causing a loss of information or accuracy.

2 Setup of Digital Design and Fabrication Processes for Vaults

This paper describes a continuous digital design workflow for masonry vaults, based on the form-finding technique Thrust Network Analysis (TNA) [7, 8]. This method allows the design of compression-only networks based on discrete force diagrams. The resulting equilibrium networks are the control geometry of continuous NURBS surfaces [9] which are used for the generation of stone block geometries, analysis data, and fabrication and construction information. In order to process these different data representations, commercial software packages and CAD programs are used, connected and extended via custom developed interfaces to increase the software's capabilities.

The sophisticated, but well-defined task of shell design allows the setup of a tight prototypical computational workflow *in vitro*. The paradigm for this approach is the work of Heinz Isler, who developed a tight design process for shells based on physical models. The consistent integration of design information with geometric data, presented in this paper, may be interpreted as project-specific Building Information Modelling (BIM) [10].

Section 3 describes how different data transitions via custom tools and interfaces are managed in detail. In section 4, a design-to-fabrication chain for structural models using rapid prototyping (3D printing), is demonstrated. It will be shown that this process is very close to a process that might be used for the actual goal of designing large-scale stone masonry vaults.



3 Customized Linking of Data and Applications

This chapter describes in detail strategies for the setup of a digital workflow for computational vault design and realization. In order to efficiently deal with the diverse challenges of form finding, analysis, and fabrication, commercial software packages are linked with applications that were developed for this research project. The link between these different programs is established by custom-made software interfaces. These interfaces provide a structured transition between different data representations of geometry and structural properties. The use of build-in import/export routines instead of custom interfaces would cause a loss of geometrical and structural information. The digital design workflow can be divided into three main phases (figure 1). Phase I represents the form-finding process of the compression-only surface based on the TNA method. In phase II, the geometry of the individual stone blocks is generated from the compression-only surface. In phase III finally, different data sets are generated, for discrete element analysis (DEA), computer numerically controlled (CNC) stone cutting, rapid prototyping, and the production of the scaffolding. These three phases happen roughly in this order in time, but with several feedback loops.

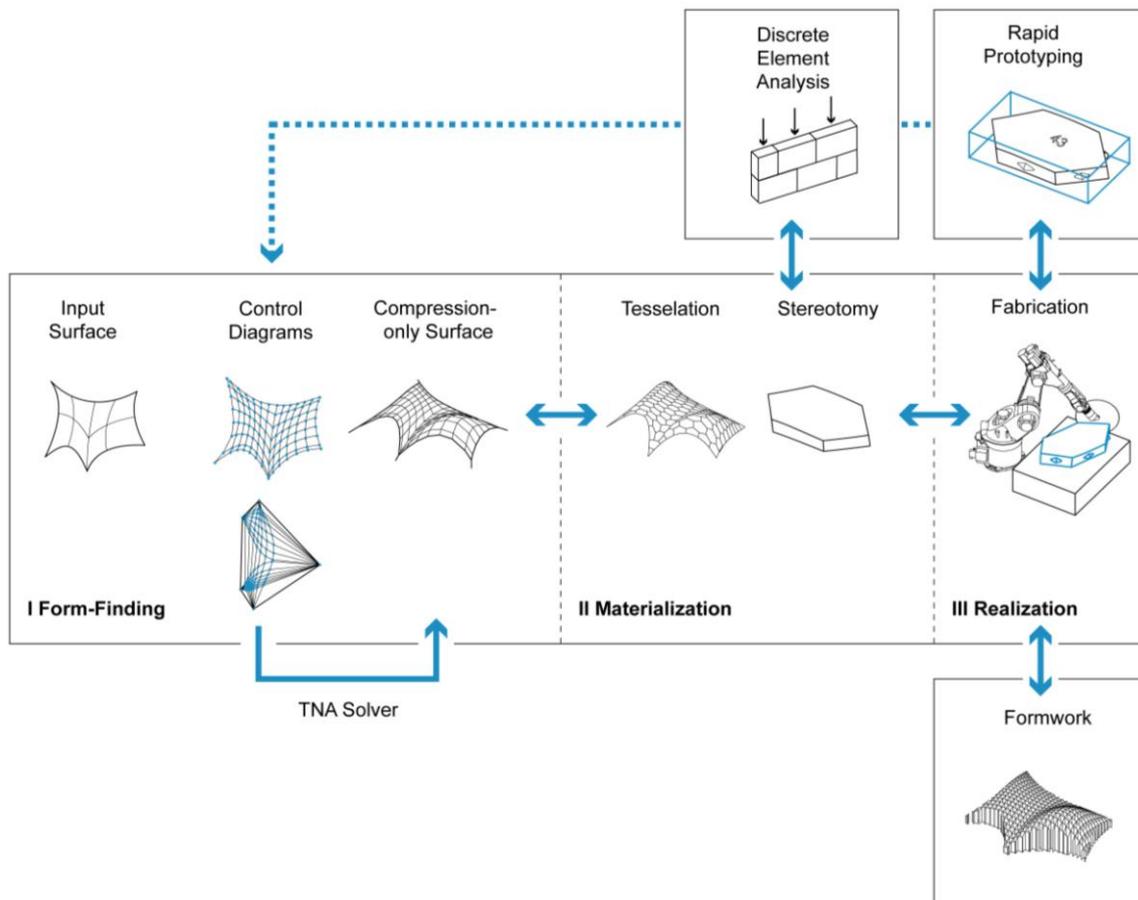


Figure 1: Digital Workflow: From Form Finding to Fabrication



3.1 Phase I: Form Finding

Thrust Network Analysis (TNA) uses projective geometry, reciprocal diagrams and linear optimization to provide a graphical, intuitive and real-time method, adopting the same advantages of techniques such as graphic statics [11], but offering a viable extension to fully three-dimensional problems.

(a) Input surface

The first step defines the general outline of the shell structure in plan. It is represented by a single or poly surface (using NURBS surfaces) in a NURBS-modeling environment, such as McNeel's Rhinoceros® [12]. Both CAD applications feature tools to easily generate and manipulate the form-determining surface. Further customized tools were developed to make the process of manipulation more intuitive for the user.

(b) Control Diagrams

By addressing the input surface's local coordinate systems the planar force network, representing possible force lines in the structure, is generated automatically. This process is fully integrated in the CAD environment interactively responding to any changes made on the input surface. "Stitching" different NURBS surfaces, with each their internal force pattern logic, allows the generation of sophisticated force line topologies.

Based on the nodes and branches of the planar force network the reciprocal force diagram is produced according to Maxwell's definition of reciprocal figures [13] through two consecutive linear optimization procedures [8]. The reciprocal diagram, visualizing the proportional relationship of horizontal forces in the network, can then be interactively manipulated by the user in order to vary the shape of the shell structure. The process of manipulating the grids is fully integrated into a live feedback-loop allowing for visual control of the resulting geometry in real-time.

(c) Compression-only surface

The output of this interactive form finding, which mediates between structural, visual and functional needs, is provided as a single or poly NURBS surface automatically generated from the point set data of the TNA. The TNA solver is integrated by implementing the matrix computation routines from Rhino SDK [14] in custom-made Grasshopper vb.net nodes [15] and by calling MatLab executables [16] from RhinoScripts [17]. All information about the grid topology is maintained by addressing the corresponding local coordinates of the *thrust surface* that represents the ideal shape for the compression-only shell geometry according to the given input parameters. Therefore, all data can be extracted and processed smoothly for the further phases of the design process.

2.2 Phase II: Materialization

The resulting thrust surface of the first phase represents an ideal compression-only form for a given, dominant loading case (which is also definable by the user), but is not yet



influenced or driven by constraints arising from fabrication and construction techniques as well as material properties. For the surface's physical representation further constraints have to be taken into account. The following steps are based on the realization of the shell's physical scale model.

This paper particularly describes the process of designing vaulted masonry-like structures. The way of tessellating the surface is of key importance for the further process. The tessellation of the geometry is influenced by the general shape of the surface, linked to the design intent, its visual appearance (tectonics), the structural analysis (force flow) and of course fabrication and material properties. The interdependency of these parameters led to an approach that allows visual feedback and multiple ways of controlling the process.

(a) Tessellation

First, a 2D topology for the given surface is defined, primarily depending on the number and proportion of open and closed edges and the number of supports (Fig. 2). This pattern is represented visually by a 2D polyline formation that can either be generated by the user or by mathematic algorithms for ruled based configurations.

Using relaxation procedures, based on the transient stiffness method [18], which are tightly integrated in the general digital workflow, the pattern is distributed evenly on the surface. This approach allows to computationally control the edge conditions. In contrast to e.g. pulling or projecting a pattern onto a surface, which typically results in problems, or special cases, at the borders and corners of the surface patch to be tessellated, this approach solves all these issues, connecting this step nicely to the smooth digital workflow. Figure 2 shows the relation of the 2D pattern and the corresponding surface boundary for different plan geometries.

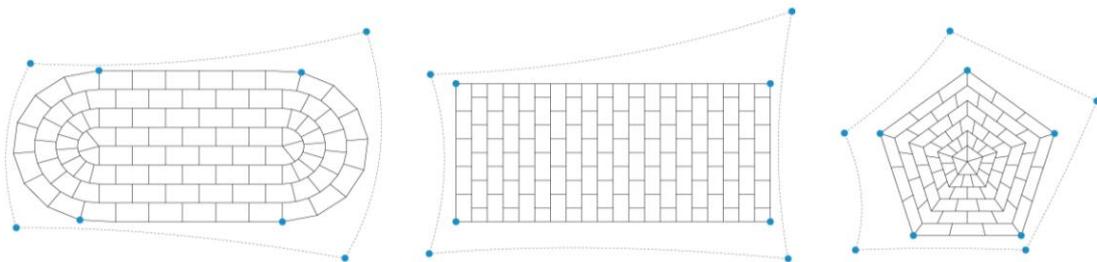


Figure 2: 2D patterns for different surface (edge) geometries.

During the iterative process of distributing the pattern evenly on the surface, all vertices stay on the surface (Fig. 3). Certain vertices are fixed at supports (the heavier dots in Fig. 2) and edge vertices are constrained to only move along the surface edge.

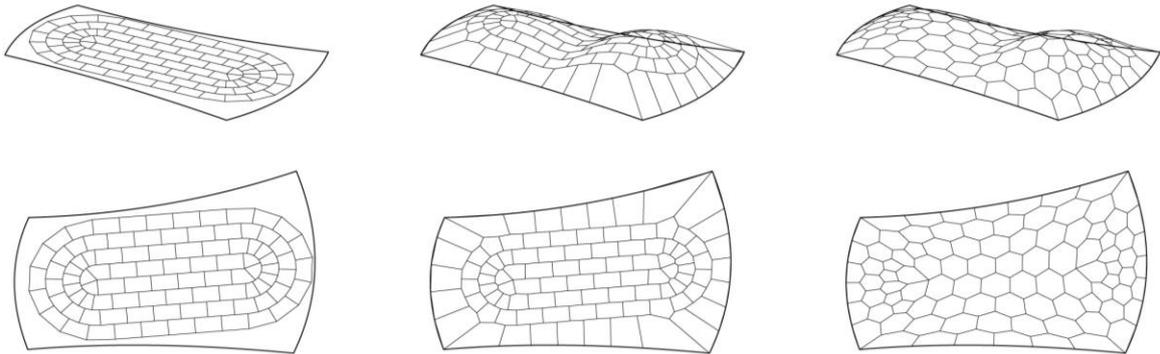


Figure 3: (a) Start configuration and relaxed pattern after (b) 5 and (c) 200 iterations.

The relaxation procedure can further be informed by restrictions arising from the force distribution, the need for interlocking or staggered tessellation and fabrication constraints like material and tooling properties (e.g. compression strength, friction, size, maximum weight per piece or planar cutting planes). One way to incorporate those requirements is to generate specific guides addressing the aforementioned aspects. For example, as shown in Figure 4, certain groups of vertices can be forced to move only along provided lines, which e.g. result from a surface analysis of the structural thrust surface. Such additional constraints result in more control of the 3D pattern.

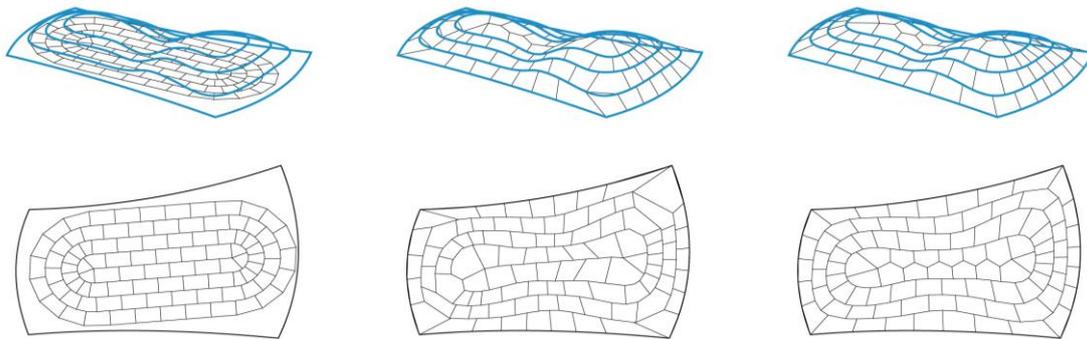


Figure 4: (a) Start configuration and relaxed pattern after (b) 5 and (c) 200 iterations with control guides.

(b) Stereotomy

Based on the tessellation pattern, individual blocks are generated using a custom RhinoScript routine. The NURBS surface provides the geometric information of the surface normals; the load assumptions, used for form finding, determine the depth of the blocks. The block interface surfaces, generated by this routine, are double curved, which is no problem for 3D printing. In order to build the vault from stone blocks, further research has to be done to simplify the geometry of the stone blocks. Planar interface surfaces e.g. would be significantly more efficient for stone cutting. For the model, additional block geometry modifications have been implemented, such as the application of notches and numbers.



2.3 Phase III: Realization

In the realization phase, different data sets are produced from the compression-only surface and the block geometries for the purpose of analysis, model production, fabrication, and assembly.

(a) Discret Element Analysis

Discret Element Analysis (DEA) is a method originally developed for rock mechanics. It is used for computational testing of the mechanical behaviour of discret elements. Recent approaches show the application of DEA for masonry structures [19]. The commercial package 3DEC [20] used in this research works with convex polyhedra as input. To generate the input data, the stone block geometries have to be subdivided in convex polyhedra. In a second step they have to be exported in the specific data format used by the software package. Both steps are implemented as software applets for Rhinoceros using RhinoScript.

(b) Rapid Prototyping

Rapid Prototyping is used to produce structural models for physical testing. Several routines were developed for labelling and registration of the pieces. Further post-processing routines for the optimization of the 3D-printing time, material and cost have been programmed (Fig. 6).

(c) Fabrication

For the fabrication of the individual stone blocks, in a first step, the pieces have to be aligned automatically, so that the cut-off is minimized. Then the fabrication data has to be produced from the block geometry, so that the CNC machine is cutting the stone in an efficient way, reducing cutting time and therewith cost.

(d) Formwork

Formwork has to be built to assemble the real-scale shell. Software routines have to be developed, to generate efficient geometry of these structures automatically. The formwork is a huge expense factor for building shells, so the focus in the optimization of the scaffolding geometry should be on cost reduction.

4 Application: Structural Model Building

This section describes the production and assembly of a structural model as an application of the digital process described in the previous section: a masonry vault, built from individually cut stone blocks. In the scale model, each stone is represented by a piece of 3D-printed resin. The design, production, assembly, and structural behaviour of the scale model have analogies to the design, fabrication, building, and behaviour of the real masonry-like vault. These analogies allow the anticipation of challenges in the planning and construction process of the real scale vault while building the structural model. Some strategies and software tools, developed for the rationalization of the



model planning, can be directly used in the fabrication process of the real vault. Analogies between scale model and real building are the followings:

4.1 Structural Behaviour

The form of the vault and the geometry of the individual stone blocks is designed, such that the vault stands without any mortar, just by friction between the blocks. This is possible, because of the existence of a compression-only force pattern, generated with TNA. The structural behaviour of such a shell structure is invariant to scaling, as long as the blocks have sufficient friction [21]. This invariance in scale allows the building of a scale model with the same structural behaviour like the real vault from any not deformable modelling material.

4.2 Logistics and Labelling

Models often consist of hundreds of pieces. To assemble them, a labelling system has been introduced, which allows the identification of a given model piece in the CAD model. Therefore a custom software applet has been developed using RhinoScript, that automatically marks the pieces in the digital model. The numbering is related to the order/sequence of assembly. The labels appear as “stamps” in the 3D-printed pieces (Fig. 5). In the real stone vault, one has to deal with similar problems of logistics and labelling. First, a sequence of construction has to be developed. The labelling system of the stone blocks has to represent construction sequence, position, and orientation of each block in the shell.

4.3 Minimization of Material

Before starting the rapid prototyping process, an important pre-processing step is the alignment of the pieces in the CAD program. A dense packing of the pieces enables a significant reduction of the resin amount, production time, and cost of the model. A custom-made software applet has been developed in order to arrange the pieces automatically. A first routine computes the minimal bounding box of each piece, and then horizontally aligns them. A second routine nests the pieces in a box representing the building space of the rapid prototyping machine (Fig. 5). For the fabrication of the real scale stone blocks, the automatic alignment according to the minimal bounding box minimizes the cut-off of stone material.

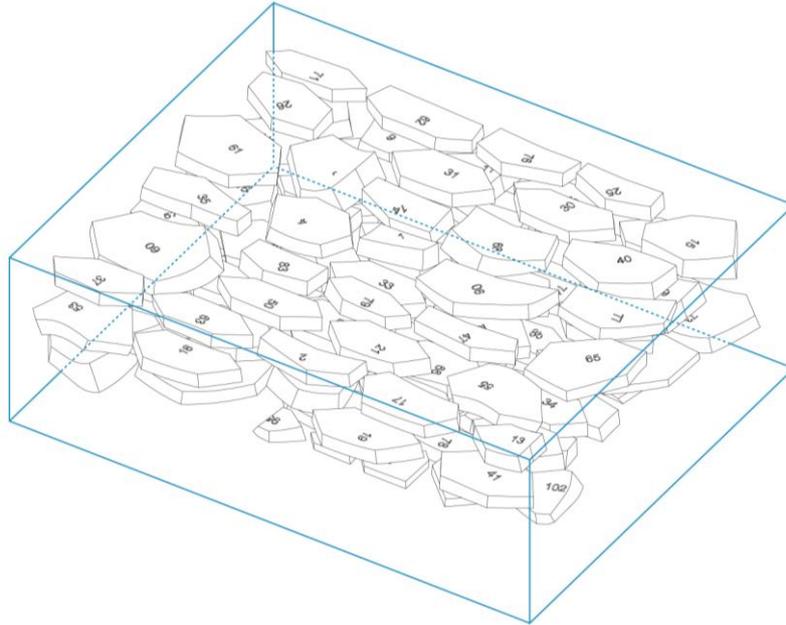


Figure 5: Nesting for rapid prototyping

4.4 Formwork

Instead of assembling the model pieces using formwork in model scale, the pieces are assembled upside-down in a bowl-like negative form of the shell. This negative form, or cradle, is cut out from a foam block with a 3 axis CNC mill. For proper registration of the pieces in the cradle, a system of notches has been developed. They were automatically generated in the digital model by a custom script (Fig. 6). After arranging all pieces in the cradle, the base plate with slots at the supports is placed on top of the foam block. Then, the whole package of cradle, base plate and pieces are flipped. Finally, the cradle is removed and the model stands just by friction between the pieces. For the building process of the stone vault, scaffolding has to be developed. Based on the same surface information as the cradle, a process for the automated generation of scaffolding geometry and then fabrication data has to be set up.

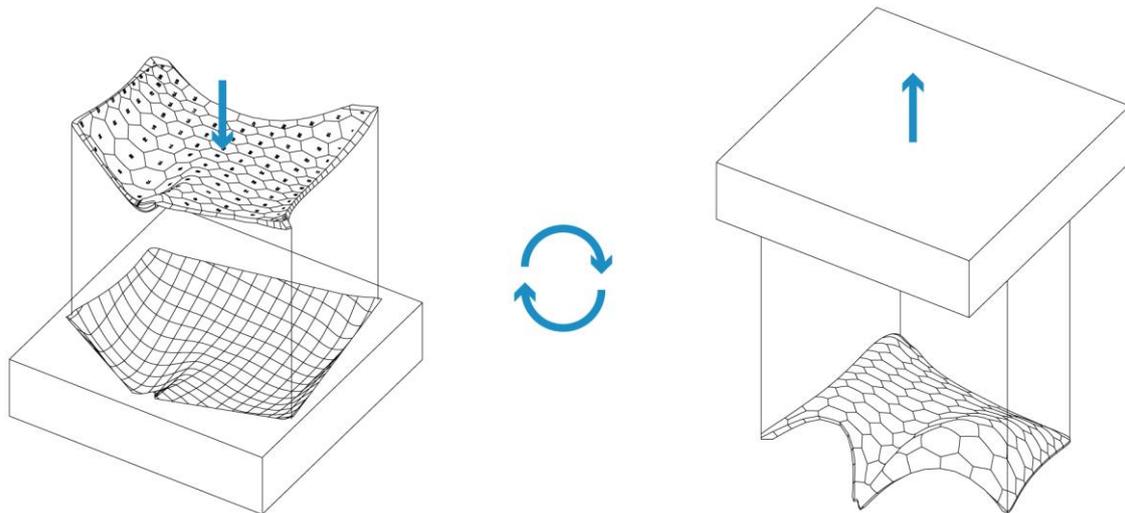


Figure 6: Cradle, model setup and assembly.

5 Conclusions and Outlook

This paper has described a powerful, interactive process for designing structural, masonry-like vaults. A smooth digital workflow has been developed, going from form finding to fabrication, dealing with a variety of constraints. The form finding is based on TNA, which uses discrete control and force diagrams, to explore structural form. These discrete networks are directly the control framework for NURBS surfaces, which contain the continuous geometrical and structural information.

In the second and third phase of the digital workflow, important, further research needs to be conducted to fully integrate fabrication constraints which are related to production efficiency and latest CNC-fabrication techniques of stone cutting. However, the developed process is already structured to easily integrate more complex parameters emerging from the abovementioned aspects.

The application of the described workflow for realizing full-scale, “free-form”, masonry-like vaults will be the next main focus of research.

References

- [1] Tomlow J, Graefe F, Otto F, and Szeemann H. *The Model*. Institut für Leichte Flächentragwerke, 1989, **34**, Stuttgart.
- [2] Gass S and Otto F. *Experiments*. Institut für Leichte Flächentragwerke, 1990, **25**, Stuttgart.
- [3] Chilton J. *Finding Heinz Isler – The Engineer’s Contribution to Contemporary Architecture*, Thomas Telford Ltd., 2000.



- [4] Dohmen P and Rüdener K. Digital Chains in Modern Architecture. In *Proceedings of eCAADe 2007 Conference*, Wiesbaden, 801-804.
- [5] Schindler C. Information-Tool-Technology: Contemporary digital fabrication as part of a continuous development of process technology as illustrated with the example of timber construction. In *Proceedings of the International ACADIA Conference 2007 Expanding Bodies*. Halifax Nova Scotia.
- [6] Kilian A. Linking Hanging Chain Models to Fabrication. In *Proceedings of the 23rd Annual Conference of the Association for Computer Aided Design in Architecture and the 2004 Conference of the AIA Technology in Architectural Practice Knowledge Community*, Cambridge, 110-125.
- [7] Block P and Ochsendorf J. Thrust Network Analysis: A new methodology for three-dimensional equilibrium. *Journal of the International Association for Shell and Spatial Structures*, 2007; **48**: 167-173.
- [8] Block P. Thrust Network Analysis: Exploring Three-dimensional Equilibrium. PhD dissertation. Massachusetts Institute of Technology, Cambridge, MA, USA, 2009.
- [9] Farin G. *Curves and Surfaces for CAGD: A Practical Guide* (5th edn.). San Francisco, Morgan Kaufmann Publishers, 2002.
- [10] Coenders JL. Parametric and associative design as a strategy for conceptual design and delivery to BIM. In *Proceedings of the International Association for Shell and Spatial Structures (IASS) Symposium 2009*, Valencia, 1112-1123.
- [11] Allen E. and Zalewski W. *Form and Forces: Designing Efficient, Expressive Structures*. New York, John Wiley Sons, 2009.
- [12] McNeel R. RHINOCEROS[®]: NURBS modeling for Windows [Computer software]. <http://www.rhino3d.com/>.
- [13] Maxwell, JC. On reciprocal figures and diagrams of forces. *Philosophical Magazine and Journal Series* 1864; **4**(27): 250-261.
- [14] McNeel R. Rhino.NET Framework SDK (Software Development Kit). <http://wiki.mcneel.com/developer/dotnetplugins>.
- [15] McNeel R. Grasshopper (generative modeling for Rhino). <http://www.grasshopper3d.com/>.
- [16] The Mathworks. MATLAB[®] CompilerTM. <http://www.mathworks.com/products/compiler/>.
- [17] Rutten D. *RhinoScriptTM 101 for Rhinoceros 4.0*. Robert McNeel Associates, 2007.
- [18] Lewis W. *Tension structures: Form and Behavior*, Thomas Telford Ltd., 2003.
- [19] DeJong, M. J., Seismic Assessment Strategies for Masonry Structures. PhD dissertation. Massachusetts Institute of Technology, Cambridge, MA, USA, 2009.
- [20] Itasca: 3DEC - Three Dimensional Distinct Element Code [Computer software]. <http://www.itascacg.com/3dec/>.
- [21] Block P, Lachauer L and Rippmann M. Validating Thrust Network Analysis using 3D-printed, structural models. *Proceedings of the International Association for Shell and Spatial Structures (IASS) Symposium 2010*, Shanghai, China.