

In this case study, we analyze the PAT Center of Richard Rogers and Ove Arups & Partners structural engineering.

The PAT Center in Princeton, NJ, USA is a single-storey building with a floor area of 3,700 m<sup>2</sup>. Specializing in design and telecommunications, PA Technology LTD wanted the most flexible and expandable space possible as a technological center that could be adapted to unpredictable space requirements. That is why the building consists of a single large room, which can be subdivided as desired. Along the central axis, there are serially distributed pylons from which the roof is suspended.



## Task 1 Calculation of the loads

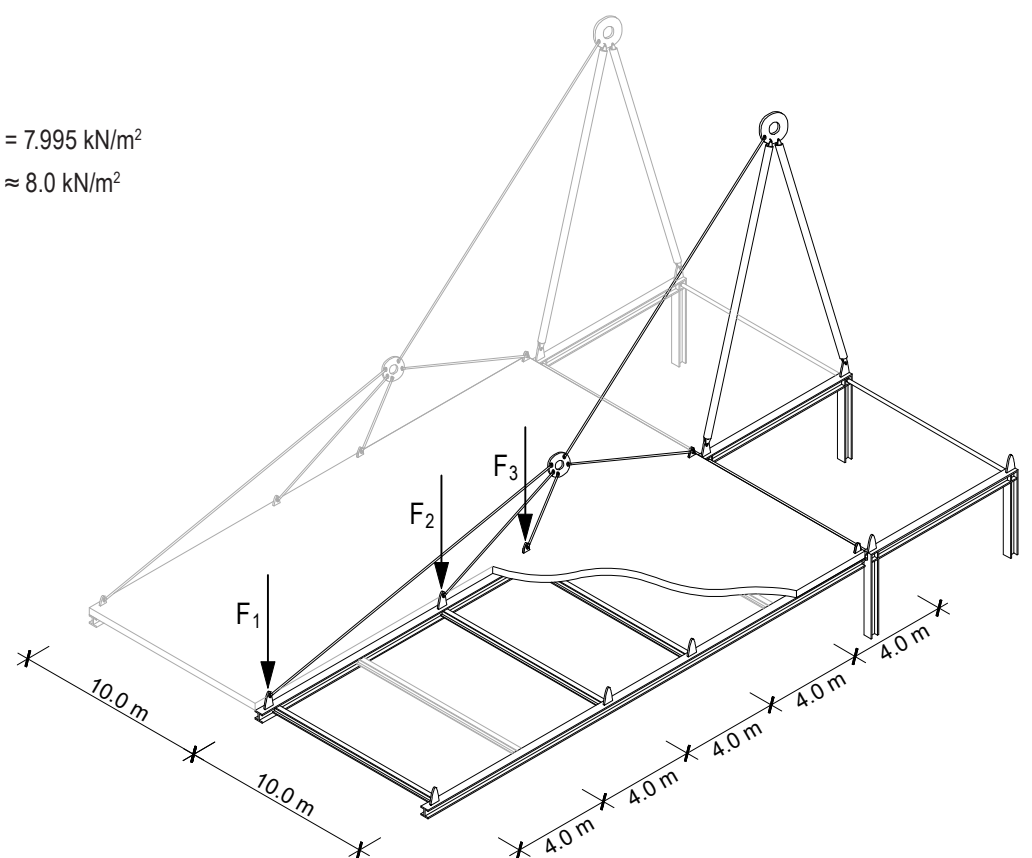
The primary longitudinal beams together with the secondary cross beams carry a ceiling of prefabricated concrete elements with a thickness of 12 cm. Concrete has a volume load  $\gamma_k$  of 20 kN/m<sup>3</sup>. On top there is the finished roof (composed by: insulation, gravel, covers) with a surface load of  $\bar{g}_k = 1.3$  kN/m<sup>2</sup>. Also it is likely to snow in Princeton in winter, so a snow load of  $\bar{q}_k = 2$  kN/m<sup>2</sup> has to be considered.

- Calculate the total area load for the concrete elements as well as the roof structure including snow at design level and round up the value to whole kN/m<sup>2</sup>.
- Calculate the point loads F1-3 acting on the primary beam with the rounded value from 1a). In the axonometric drawing, the respective dimensions of the load-influencing zones can be recognized. Important: Two bars are neglected because their weight is not transferred directly by a rope.

$$1a) \quad \bar{s}_{k \text{ concrete}} = \gamma_k \cdot d = 20 \text{ kN/m}^3 \cdot 0.12 \text{ m} = 2.4 \text{ kN/m}^2$$

$$\begin{aligned} \bar{s}_d &= (\bar{s}_{k \text{ concrete}} + \bar{g}_k) \cdot 1.35 + \bar{q}_k \cdot 1.5 \\ \bar{s}_d &= (2.4 \text{ kN/m}^2 + 1.3 \text{ kN/m}^2) \cdot 1.35 + 2 \text{ kN/m}^2 \cdot 1.5 = 7.995 \text{ kN/m}^2 \\ \bar{s}_d &\approx 8.0 \text{ kN/m}^2 \end{aligned}$$

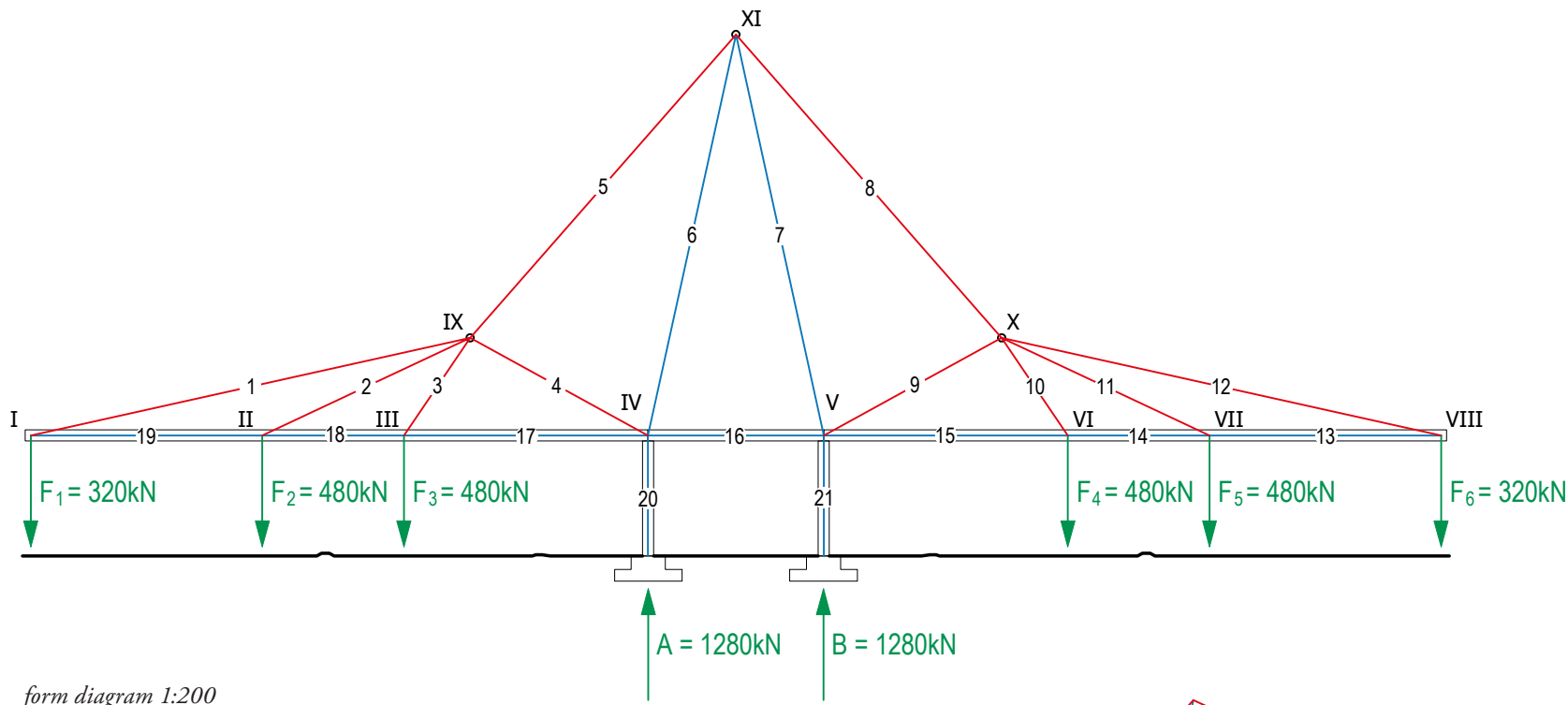
$$1b) \quad \begin{aligned} F_{1,6} &= \bar{s}_d \cdot b \cdot t_1 = 8.0 \text{ kN/m}^2 \cdot 10 \text{ m} \cdot 4 \text{ m} = 320 \text{ kN} \\ F_{2,3,4,5} &= \bar{s}_d \cdot b \cdot t_2 = 8.0 \text{ kN/m}^2 \cdot 10 \text{ m} \cdot 6 \text{ m} = 480 \text{ kN} \end{aligned}$$



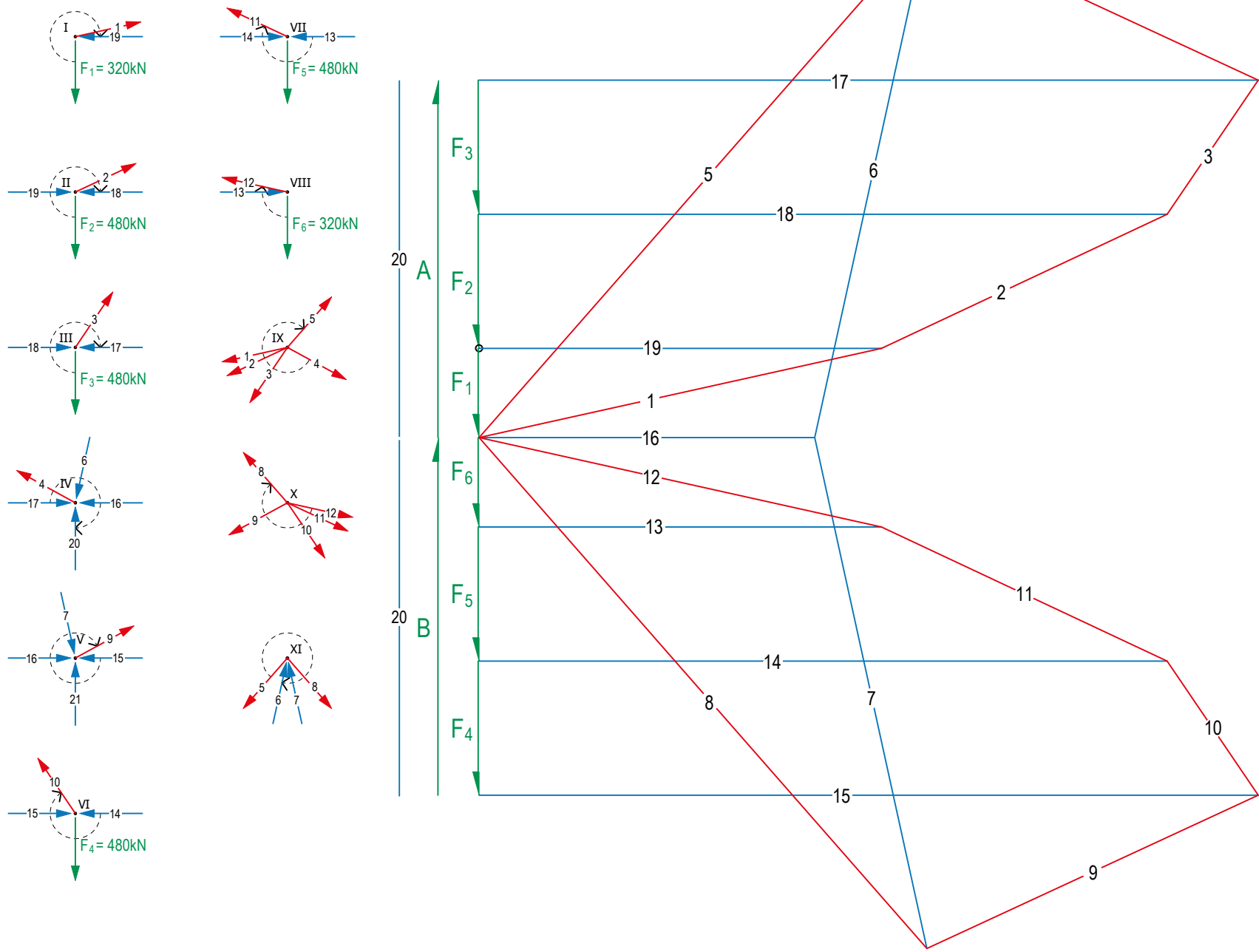
Task 2 Forces in the section

The primary structure consists of steel cables, steel tubes for the pylons and I-beams for the remaining pressure elements.

- a) The cross-section of the building is shown below in the form diagram. Draw the corresponding force diagram. Use the point loads  $F_{1-3}$  from 1b). Mark tension elements red and compression elements blue.
- b) For the supporting elements rope, deck and pylon, indicate the position and load of the relevant segments.



form diagram 1:200



subsystem

force diagram 1cm ≙ 200kN

	segment	load
rope	5 / 8	2431.2 kN
deck	15 / 17	2790.4 kN
pylon	6 / 7	1872.2 kN

### Task 3 Dimensioning the main elements

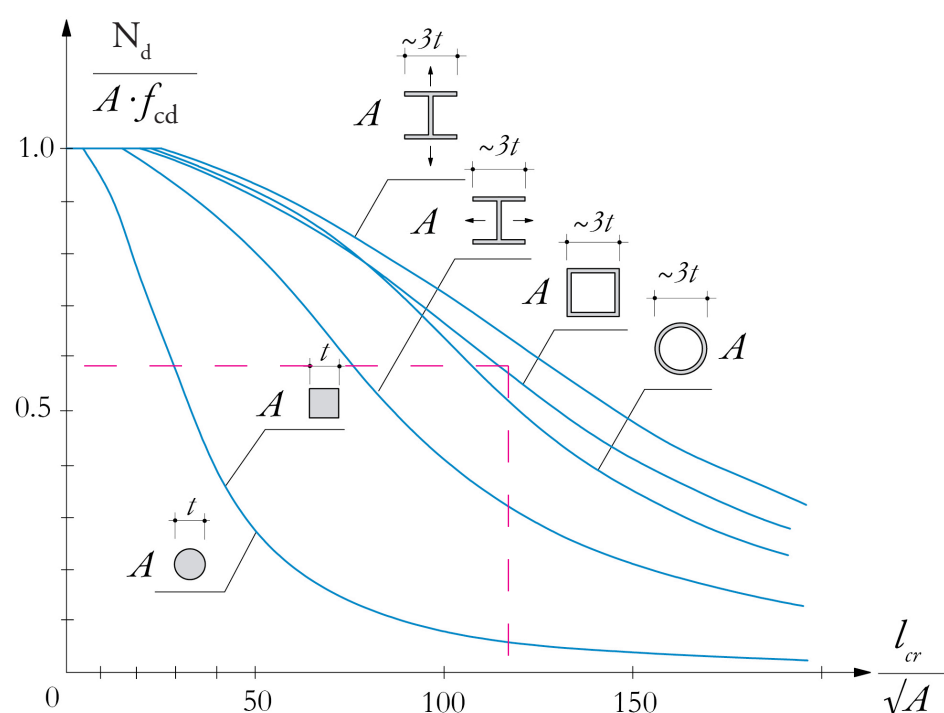
- a) Check whether the rope  $D=100\text{mm}$  made of steel S355 can withstand the maximum tensile load from task 2.
- b) The pylon shall be a round hollow section of S355 steel with an outer diameter  $D_1=160\text{ mm}$  and an inner diameter  $D_2=116\text{ mm}$ . Check on the basis of the diagram below whether this cross-section would buckle under the maximum compressive stress from task 2. (The respective lines in the diagram mark the critical limit of the buckling. If a point is below it, the segment will not buckle under the given load.  $l_{cr}$  means the length of the segment. How could a possible buckling be prevented?
- c) The main beam is constructed with the shown steel profile made of steel S500. The flange thickness corresponds to that of the web which is 35 mm. Verify that this cross-sectional area is sufficient to absorb the maximum compressive stress of the deck as determined in task 2.

$$\begin{aligned}
 3a) \quad N_d / A_{ef} &\leq f_{cd} \\
 N_{d \text{ rope}} &= 2431.2 \text{ kN} \\
 A_{ef} &= (50 \text{ mm})^2 \cdot \pi = 7853.98 \text{ mm}^2 \\
 f_{cd} &= 355 \text{ N/mm}^2 / 1.05 = 338.1 \text{ N/mm}^2 \\
 N_d / A_{ef} &= 309.55 \text{ N/mm}^2 \leq f_{cd} \\
 309.55 \text{ N/mm}^2 &\leq 338.1 \text{ N/mm}^2
 \end{aligned}$$

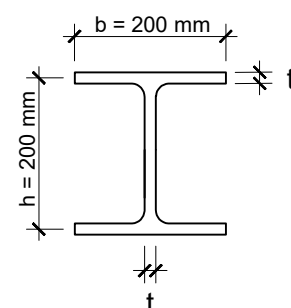
$$\begin{aligned}
 3b) \quad A &= r_1^2 \cdot \pi - r_2^2 \cdot \pi = 20106.19 \text{ mm}^2 - 10568.32 \text{ mm}^2 = 9537.87 \text{ mm}^2 \\
 N_d / (A \cdot f_{cd}) &= 1872.2 \text{ kN} / (9537.87 \text{ mm}^2 \cdot 338.1 \text{ N/mm}^2) = 0.581 \\
 l_{cr} / \sqrt{A} &= 11.6 \text{ m} / 97.662 \text{ mm} = 118.78
 \end{aligned}$$

The determined point is above the critical limit for the proposed circular profile.  
This would buckle under the given load. However, it would be possible to select a I-beam that is stressed in the direction of the web.

$$\begin{aligned}
 3c) \quad A_{web} &= 35 \text{ mm} \cdot 165 \text{ mm} = 5775 \text{ mm}^2 \\
 A_{flange} &= 35 \text{ mm} \cdot 200 \text{ mm} \cdot 2 = 14000 \text{ mm}^2 \\
 A_{ef} &= 19775 \text{ mm}^2 \\
 N_d / A_{ef} &\leq f_{cd} \\
 N_{d \text{ deck}} &= 2790.4 \text{ kN} \\
 f_{cd} &= 500 \text{ N/mm}^2 / 1.05 = 476.2 \text{ N/mm}^2 \\
 N_d / A_{ef} &= 141.11 \text{ N/mm}^2 \leq f_{cd} \\
 141.11 \text{ N/mm}^2 &\leq 476.2 \text{ N/mm}^2
 \end{aligned}$$



buckling diagram

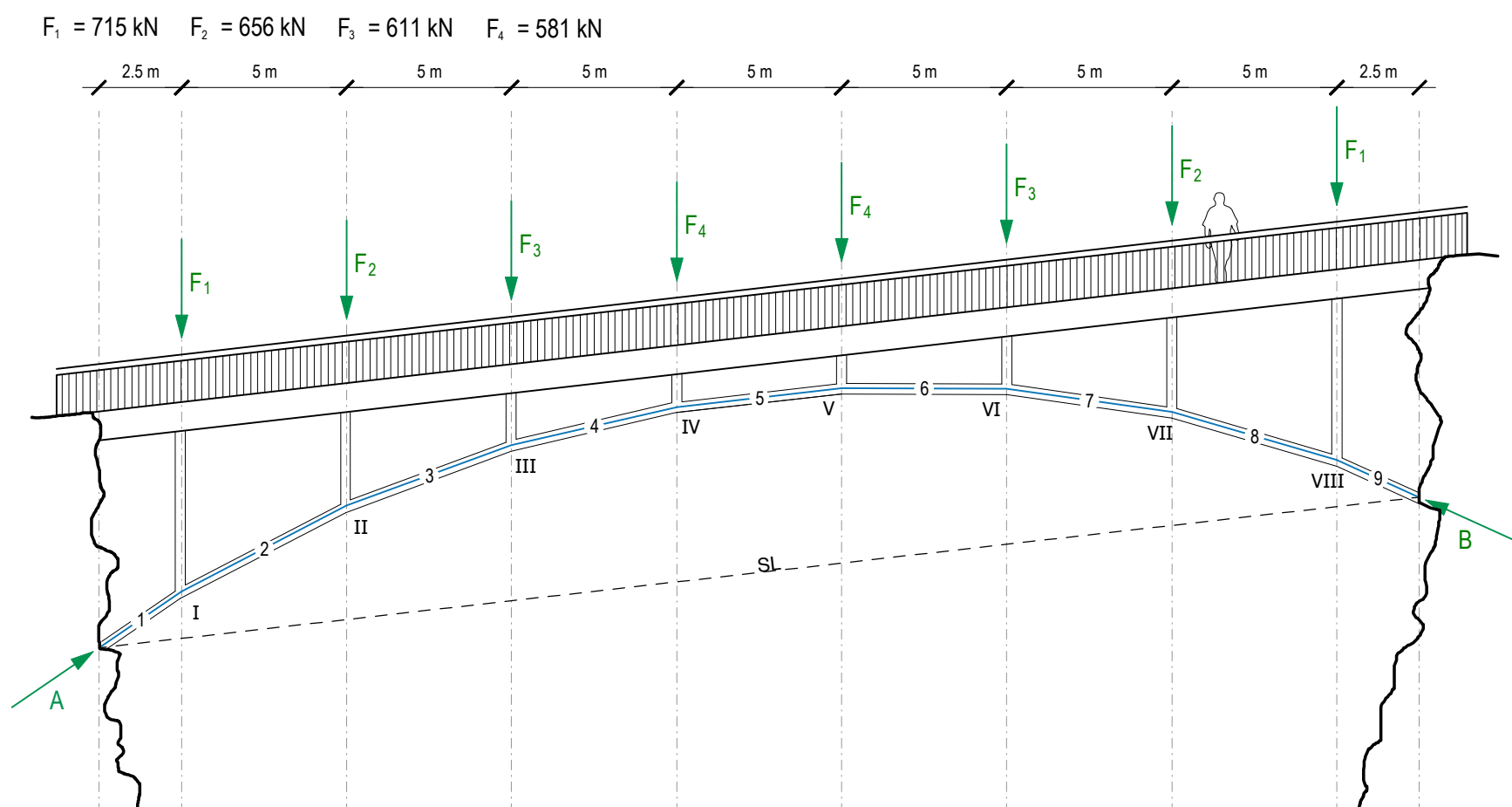


section main beam

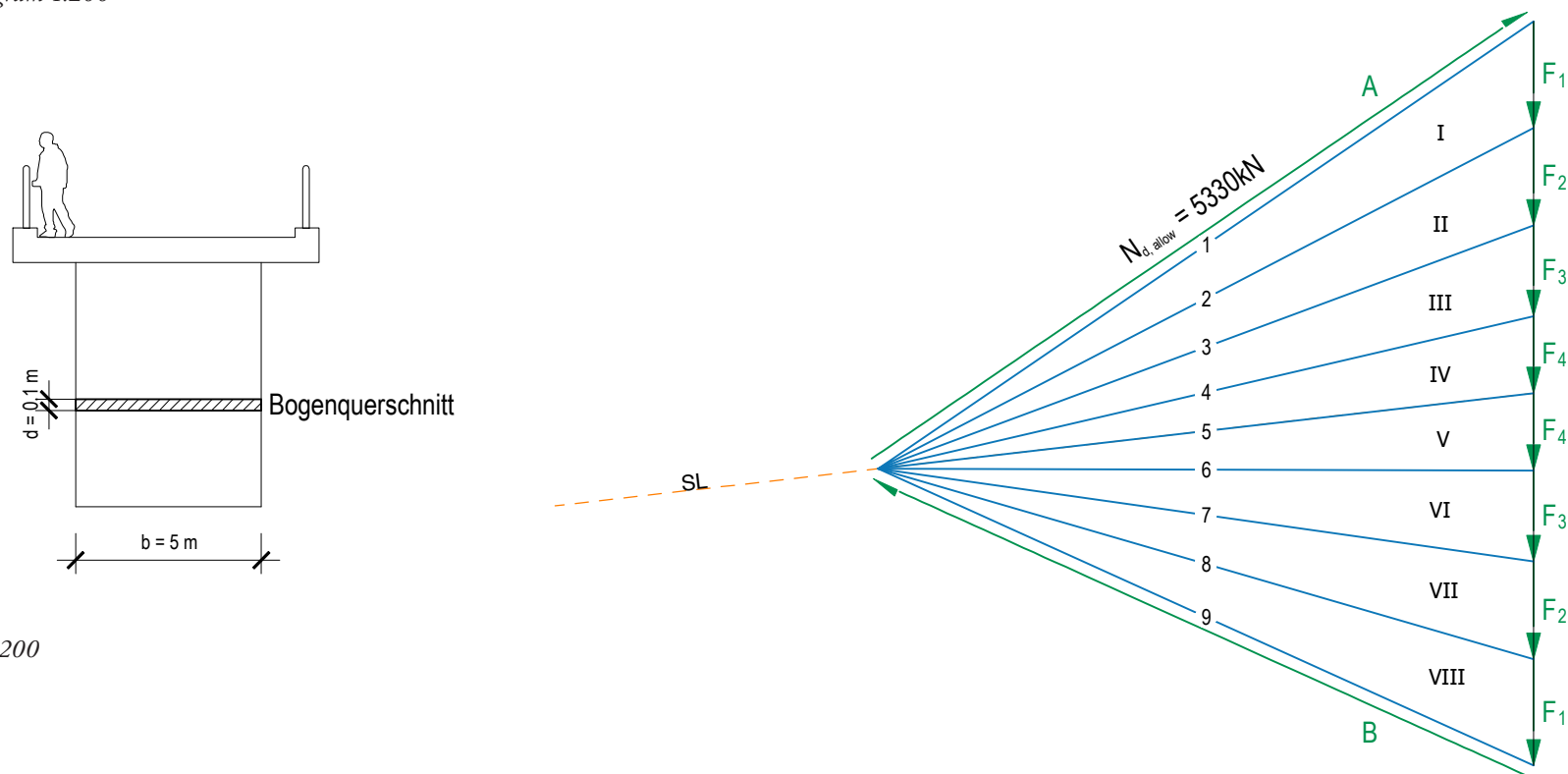
## Additional Concrete arch bridge

## Task 1

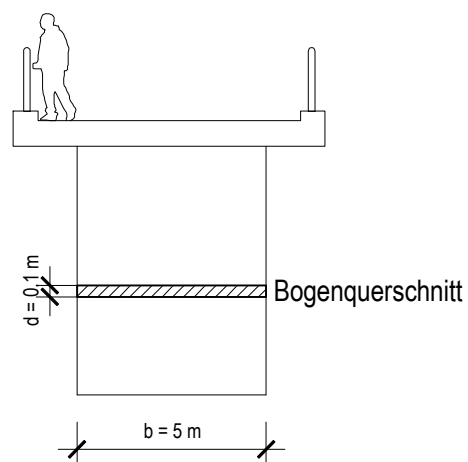
- a) Calculate  $N_{d,max}$  with a characteristic stress of  $f_{ck} = 16 \text{ N/mm}^2$  in compression and the resistance coefficient  $\gamma_m = 1.5$ . The bridge is made out of concrete C 16/20. The dimensions of the arch's cross-section are:  $d=0.1\text{m}$  and  $b=5.0\text{m}$ .
- b) Find the funicular form of the arch in pure compression and give the size, location and direction of the reaction forces. The middle segment of the arch should be parallel to the deck.



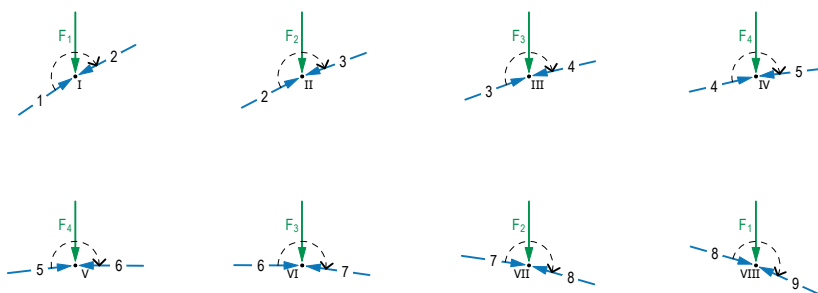
form diagram 1:200



section 1:200

force diagram 1cm  $\hat{=}$  500kN

$$\begin{aligned}
 a) \quad A_{eff} &= b \cdot d = 5000 \text{ mm} \cdot 100 \text{ mm} = 500'000 \text{ mm}^2 \\
 f_{cd} &= f_{ck} / \gamma_m = 16 \text{ N/mm}^2 / 1.5 = 10.66 \text{ N/mm}^2 \\
 N_{d,allow} &= f_{cd} \cdot A_{eff} = 10.66 \text{ N/mm}^2 \cdot 500'000 \text{ mm}^2 = 5330 \text{ kN}
 \end{aligned}$$



## Additional Task 2 Estádio Municipal de Braga by Eduardo Souto de Moura

- a) Precast concrete panels are placed over both of the two 55m wide terraces. Every concrete panel with a width of  $b=3.7\text{m}$  and a thickness of  $d=240\text{mm}$  is fixed on two hanging cables. Calculate the uniform line load  $g_d$  per cable at measuring level and enter the value in  $\text{kN/m}$ . Spatial load reinforced concrete  $\gamma_k=25\text{kN/m}^3$
- b) After a heavy snowfall the snow only remained on the north side of the roof whilst it melted on the southern side. This leads to asymmetrical loads. Calculate the live load  $q_d$ . Area load snow  $\bar{q}_k = 5.4 \text{ kN/m}^2$
- c) Find the static form of the hanging cable which spans between the supports A and B. The maximum cable strength is  $2'454 \text{ kN}$ . Draw the form and force diagram. How big are the reaction forces A and B?
- d) Dimension the cross-section of the cable if steel S500 is used. Enter the diameter D rounded off to mm.

$$\begin{aligned} \text{a) } A &= l \cdot b = 3.7 \text{ m} \cdot 0.24 \text{ m} = 0.89 \text{ m}^2 \\ g_k &= A \cdot \gamma_k = 0.89 \text{ m}^2 \cdot 25 \text{ kN/m}^3 = 22.2 \text{ kN/m} \\ g_d &= g_k \cdot \gamma_G = 22.2 \text{ kN/m} \cdot 1.35 = 30.0 \text{ kN/m} \\ g_d \text{ p. cable: } &30.0 \text{ kN/m} / 2 = 15 \text{ kN/m} \end{aligned}$$

$$\begin{aligned} \text{c) } R_L &= g_d \cdot l = 15 \text{ kN/m} \cdot 55 \text{ m} = 825 \text{ kN} \\ R_R &= g_d \cdot q_d \cdot l = 2 \cdot 15 \text{ kN/m} \cdot 55 \text{ m} = 1'650 \text{ kN} \end{aligned}$$

$$\begin{aligned} \text{b) } q_k &= \bar{q}_k \cdot l = 5.4 \text{ kN/m}^2 \cdot 3.7 \text{ m} = 20 \text{ kN/m} \\ q_d &= q_k \cdot \gamma_Q = 20 \text{ kN/m} \cdot 1.5 = 30 \text{ kN/m} \\ q_d \text{ p. cable: } &30.0 \text{ kN/m} / 2 = 15 \text{ kN/m} \end{aligned}$$

$$\begin{aligned} \text{d) } N_d &= 2'454 \text{ kN} \\ f_{td} &= f_{tk} / \gamma_M = 500 \text{ N/mm}^2 / 1.05 = 476.2 \text{ N/mm}^2 \\ A_{\text{req}} &= N_d / f_{td} = 2'454 \text{ kN} / 476.2 \text{ N/mm}^2 = 5'153 \text{ mm}^2 \\ D &= \sqrt{4 \cdot A / \pi} = \sqrt{4 \cdot 5'153 \text{ mm}^2 / \pi} = 82 \text{ mm} \end{aligned}$$

