Prototype of an ultra-thin, concrete vaulted floor system

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Abstract

The structure of the concrete floor presented in this paper consists of a thin funicular vault, quadrilateral in plan, supported on its four corners, and stiffened by a system of "ribs" walls on its extrados. The structural prototype is completed with tension ties, which link the supports and absorb the horizontal thrusts of the funicular shell. It is a prototype for the NEST-HiLo project to be realized late 2015 in Dübendorf, Switzerland.

The solution is inspired by built examples in tile vaulting in which thin vaults are stiffened by diaphragms, also called spandrel walls. In the presented structure, this structural system is implemented and constructed in high-performance concrete to achieve an extreme thinness (2 cm in the case of this prototype for both vault and ribs) and to resist asymmetrical loading. This floor would thus save more than 70% of weight compared to traditional, 25-30 cm thick concrete floor slabs used in the construction of framed buildings. This directly lowers the requirements for the foundations (often a dominant resource and cost factor), but also enables lightweight building extensions and a reduction of total floor height, offering possibilities to address, among other issues, the vertical densification of cities.

A form-finding and analysis procedure for the design of such floor systems is presented, which consists of consecutive topology, shape and size optimizations.

Keywords: Funicular vaulted floor system, form finding, thin concrete shell, structural optimization.

1. Introduction

The floor prototype as well as the form-finding and analysis procedure presented in this paper are intended to further inform and develop the design of the NEST-HiLo floor. HiLo is a research & innovation unit for NEST demonstrating ultra-lightweight construction. It is planned as a 16 m×9 m duplex penthouse apartment for visiting faculty of Empa and Eawag (Figure 1).

Figure 1. Visualization of the preliminary design for NEST, with HiLo constructed at the top front corner.
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NEST is a flagship project of Empa and Eawag in collaboration with the ETH Domain. It is a dynamic, modular research and demonstration platform for advanced and innovative building technologies on the Empa-Eawag campus in Dübendorf, Switzerland, to be completed in 2015. As a "future living and working lab", NEST consists of a central backbone and a basic grid to accommodate exchangeable living and office modules, such as HiLo, allowing novel materials and components, and innovative systems to be tested, demonstrated and optimized under real-world conditions. HiLo is a collaborative effort of the BLOCK Research Group and the Assistant Professorship of Architecture & Sustainable Technologies (SuAT), both at the Institute of Technology in Architecture, ETH Zurich, joined by Supermanoeuvre in Sydney and Zwarts & Jansma Architects in Amsterdam. HiLo introduces several innovations, and this paper discusses in particular the development of nearly 50 m^2 of lightweight floor system, featuring funicular vaulting and structural ribs.

The solution is inspired by built examples in thin-tile vaulting in which thin compression vaults are stiffened by diaphragms, also called spandrel walls. This building technique has a long tradition in Spain, but was also exported at the end of the nineteenth century to the United States by Rafael Guastavino, who developed many patents based on different possibilities of the technique (Figure 2.a) (Ochsendorf [5]).

In traditional constructions, spandrel walls had two main objectives: reach the horizontal position to place the floor finishing and contribute to the structural behavior to maintain a minimum thickness of the vault while resisiting assymetrical loading. Contemporary projects like the SUDU prototype in Addis Ababa, Ethiopia have also applied this technique successfully, combining a thin unreinforced masonry vault and spandrel walls (Figure 2.b) (Davis and Block [3]).

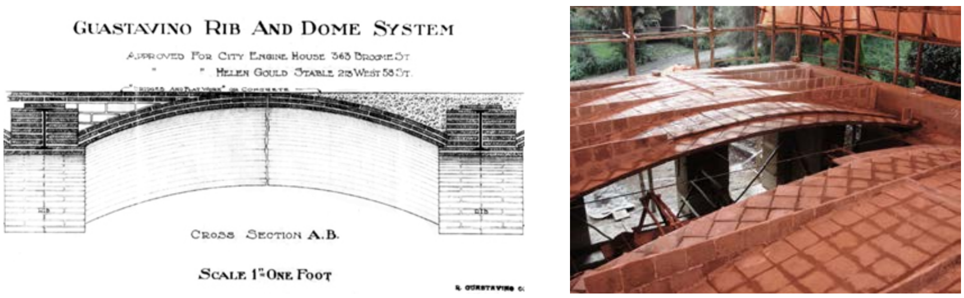

Figure 2: a) The "Guastavino Rib and Dome System",
b) the thin-tile floor of the SUDU project in Addis Ababa Ethiopia, 2010.

The structure of the ultra-thin, concrete vaulted floor system has a quadrilateral shape in plan with two adjacent square angles, and is supported on its four corners. Both vault and ribs are 2cm thick. The vault rises 13cm for an average span of 2.75 m. The ribs have a height that ranges between 14 cm at the supports and 2cm at the center of the vault (Figure 3).

Form finding was used for the initial design to optimize the layout of the ribs and the shape of the vaulted shell. The objective was the achievement of a compression-only shell under self-weight and dead load, combined with an optimized amount and arrangement of thin ribs for the remaining live load combinations taking away the need for conventional rebar entirely.

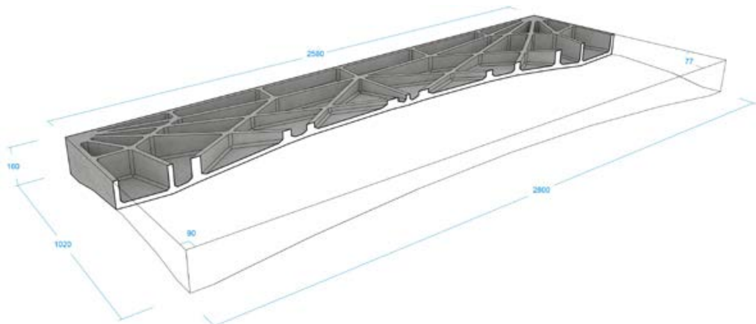

Figure 3. Perspective section of the structural prototype of the proposed floor system with units in mm.

The design process includes three steps, corresponding to the typical types of structural optimization: topology, shape and size optimization (Adriaenssens *et al*. [1]). Form finding was used for the first two steps (Section 2), whereas structural analysis was used for the third one (Section 3).

New strategies that combine form-finding methods with structural analysis, have been implemented, while using innovative approaches of design control (Figure 4). RhinoVAULT allows the creation of compression-only surface structures (BLOCK Research Group [2], Rippmann *et al*. [7]). Using new extensions of the software (Ripmann and Block [8]), it was possible to account for the non-uniform self-weight of the floor system and include the additional dead load of the finished floor above. Furthermore, these new capabilities of RhinoVAULT allowed constraining the vault's thrusts to accommodate the desired boundary conditions (Section 2). Finite element analysis was used to perform structural assessment according to Swiss building codes (SIA 261). Different material properties, thicknesses, boundary conditions and load combinations were checked in order to reproduce the several choices in the characteristics of the elements and the diversity of possible states of the structure regarding construction process and loading hypotheses (Section 3).

CNC milling and wire cutting techniques have been used to produce the double-sided mold used to cast the structural prototype of the floor system using a high-performance, flowable, steel-fiber reinforced concrete (Section 4).

2. Form-finding process

The first two steps of the design process are aimed at optimizing the topology of the pattern of the ribs, then the shape of the vault.

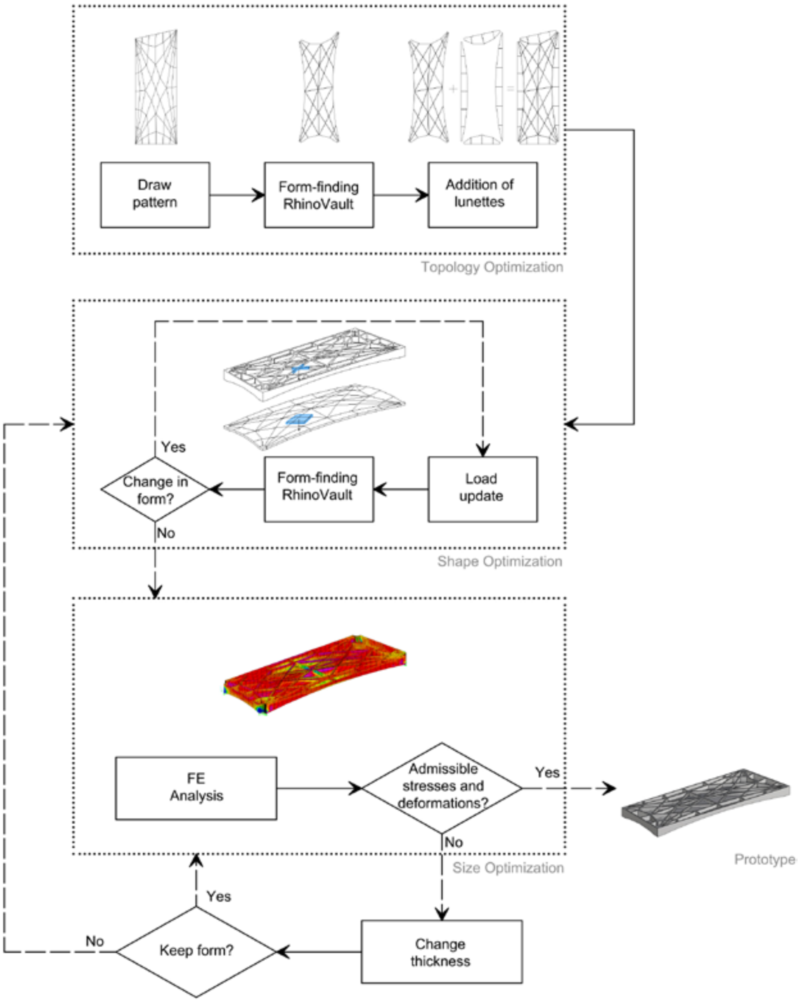

Figure 4: Flowchart for the three optimization steps.

2.1. Topology optimization

The pattern of the ribs is inspired by ribbed masonry vaults built hundreds of years ago, and takes into account the expected force flow to the supports. This first input influences the structural behavior of the vault, so it needs to be modified during the form-finding process with RhinoVAULT in order to achieve the best compression-only solution with an optimized spatial force flow. However, as edge arches of typical compression-only solutions are inclined to the inside of the vault, a strategy had to be found to accomplish a quadrilateral shape (with straight edges) in plan. Many strategies have been used in the past to fit domed or vaulted surfaces in polygonal or quadrilateral shapes in plan. One of them was the addition of lunettes, which are arching apertures in a wall or vaulted ceiling. These could additionally stiffen the structure. Lunettes were added to the four edges of the vaulted prototype (Figure 4) with the added purpose that they have load-bearing capacity and are stable by themselves.

2.2. Shape optimization

The shape optimization was performed using an iterative process of form finding in RinoVAULT in which loads are updated at each step. The starting geometry of the shell was calculated based on the self-weight of the vault with a constant thickness. Then, the weight of the ribs and finishing materials of the floor were taken into account for a new form-finding step. The weight of the ribs was calculated from the local depth, thickness and density of concrete; that of the floor finishings from the tributary areas and dead load. These were then added to the self-weight of the shell. Form finding was performed using these new values for each node at each iteration until the changes in the geometry were sufficiently small.

3. Structural analysis

The last step of the design process optimizes the sizing (thickness) of the vault and ribs, by parametrically assessing the shape with Finite Element Analyis (FEA).

3.1. Criteria

The objective of the size optimization was to find the system that would use the least amount of material, i.e. the one with thinnest elements. The criteria to define a valid system was the admissibility of the deformations and stresses. Deformations should not be higher than 1/500 of the span in the serviceability limit state and stresses should not surpass the yield strength of the material in the ultimate limit state.

Masonry or concrete vaulted shells normally have low compressive stresses compared to the yield strength of the material, though special attention should be paid to specific areas such as at the point supports. Therefore, the main criterion in this case was the admissibility of the tension stresses with respect to the concrete tensile strength. This takes into account that the concrete has no reinforcing bars, assumes that the steel fibers add no significant tension capacity and avoid cracking of the concrete during the process of curing.

3.2. Material properties

The fiber-reinforced concrete used to build the prototype has not yet been load tested and the structural analysis was done before the prototype was fabricated. Trials with different concrete recipes had to be implemented before casting the double-sided mould to achieve the fluidity needed to cast the 2 cm thick ribs. Therefore, properties of a regular concrete obtained from the literature were used to perform the analysis, being aware of the necessity to use a stronger concrete with similar density when fabricating the model, which would assure an even a better behavior than the one shown in the analysis. Future work will include the testing of material samples of the floor prototype to update the data of the properties in the current models.

The properties of the concrete used to carry out the structural analysis with FEA are:

- density: 24 kN/m^3
- modulus of elasticity: 34,000 MPa
- Poisson's ratio: 0.2
- coefficient of thermal expansion: $9.9 \cdot 10^{-6}$
- compressive strength: 30 MPa

A tensile strength of 5% of the compressive strength is assumed, so 1.5 MPa.

3.3. Loads and boundary conditions

Dead loads, live loads and the self-weight of the structure were taken into account according to the Swiss building codes. The dead load considered for the deck of the system was 1 kN/m^2 and the live load 2 kN/m^2. Combinations of additional point loads of 2 kN were applied at the ribs intersection in order to check for the appearance of tensile stresses or too large deformations. A 2 kN/m^2 live load on half of the deck's surface was also defined as a load case in order to check the floor's behavior under asymmetric loading.

Combinations of these loads were applied with safety factors of 1.35 for dead loads and 1.5 for live loads.

The floor structure is supported in its four corners. For the dead load case, the structure was analyzed with its corners only pinned at the bottom, such that the top is free to deform inwards when the vault is uniformly loaded, thus avoiding tensile stresses at the supports.

3.4. Analysis results

The analysis was performed with different thicknesses of vault and ribs. It showed admissible deformations (less than 1/500 of the span) and admissible compressive and tensile stresses overall for a thickness of 2 cm (Figure 5).

It is worth pointing out a high stress concentration observed at the supports. The way the model and the boundary conditions are defined can be considered the reason for these unrealistic peak stresses. In the real prototype, the ribs will land on a solid piece of concrete forming the corner, which will distribute the stresses, hence avoiding the situation that the model shows.

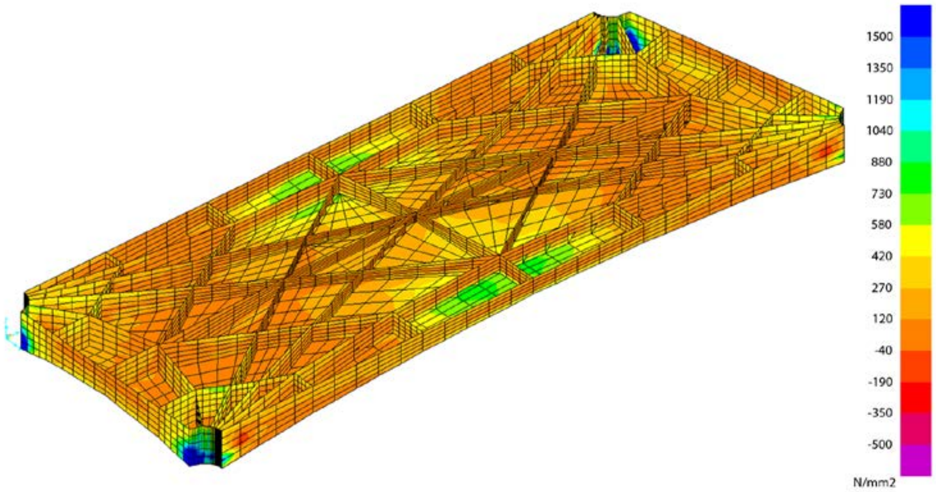

Figure 5: Stresses on the model.

3.5. Special loading cases and boundary conditions

Loading states and/or boundary conditions may also vary a lot during the construction process. Therefore, all the different stages of fabrication and assembly should be checked in terms of structural stability (Figure 6).

Before the entire structure is completed, the module will have to stand on its four supports without any frame or tension tie that can absorb its horizontal thrusts (Figure 10). The stresses affecting the structure in this state should also be checked. A model with sliding supports (not taking any horizontal thrust) and subjected only to its self-weight was therefore also defined. The tensile stresses did not even reach 0.3 MPa (=1% of the compressive strength).

The need to transport the prototype also requires specific loading cases and boundary conditions in order to verify the possibility of lifting it from points other than the supports. Two more supports were added to the model inwards of the supports along the two longer edge arches. This model under its self-weight performes satisfactorily. In case that the forces to lift the prototype would not be equally distributed between the supports, a new loading case was defined applying loads upwards in the center of the two longer edge arches. The load applied was the addition of the reaction at those points in the previous mentioned model (0.65 kN) plus 1 kN. Also in this case, no inadmissible stresses were reported.

In addition, analysis of models with four supports in different points of the model, from where the floor system could be lifted, were also performed, showing admissible stresses and deformations.

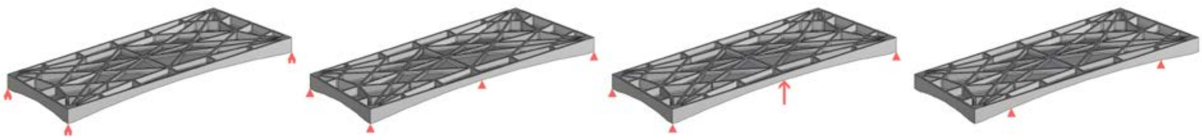

Figure 6: Special boundary conditions, considering different lifting and handling scenarios.

3.6. Verification of the system vault+ribs

To understand the contribution of each element to the system and the way they complement each other, two other models were tested: the vault without ribs and the ribs without the vault (Figure 7). Both of them were defined with the same thicknesses, material properties, boundary conditions and load cases and combinations explained above.

Both elements individually respond relatively well to distributed loads, although with higher stresses than the assumed tensile strength of the material; however, in any load combination including point loads, tensile stresses increase dramatically. The vault-only model responded slightly better to the distributed load, while the ribs-only model had a much better response against point loads. The combination of both of them gave satisfactory results, which validated the concept of the floor system.

	Vault-only	Ribs-only	Combined
Distributed load	2.9	3.3	0.8
Point load	17.7	9.7	1.5

Figure 7: Comparison of the maximum tensile stresses [MPa] for the vault-only, ribs-only and vault+ribs situations. The assumed tensile strength of the material = 5% *f_c = 1.5 MPa.

3.7. Tension ties

The horizontal thrusts transmitted from the floor system to the supports are counteracted by four tension ties connecting the supports (not shown in Figure 10). The force calculated in each of them is 27 kN in the longer edges and 12 kN in the shorter ones.

4. Fabrication of the prototype

4.1. Concrete

The concrete for the prototype is a high-performance, self-compacting fiber-reinforced concrete (SCFRC), designed to exhibit high compression strength and remain flowable despite containing steel fibers. High compression strength exploits the fact that the vault predominantly works in compression. Although the design and analysis of the floor system assumed the concrete to be unreinforced, steel fibres were added to improve performance during transport and handling, deal with stress concentrations, controll crack growth, and generally enhance flexural performance. Flowability was an important criterium due to the small dimensions of the cast. The recipe for the concrete was adapted from the second mix in Grünewald *et al*. [4]:

- 1 kg cement (Holcim Normo 5R, CEM I 52.5)
- 0.07 kg microsilica (Elkem grade 971-U)
- 1.23 kg aggregate 0/4mm
- 0.25 kg water
- 0.023 kg plasticizer (BASF Glenium ACE 30)
- 0.085 kg steel fibres (microfibres 12mm)

Average density of the concrete is 2,427 kg/m^3. The cast required 319 kg, or 0.13 m^3 of concrete. The plan area of the floor is 2.67 m^2, meaning the floor weighs an average of 119 kg/m^2. In Grünewald *et al*. [4], the 56-day compression strength is reported to be 144,7 MPa, while splitting tensile strength is 16,4 MPa. Based on two samples from the prototype, the 42-day tensile bending strength of the floor system's concrete is 14.2 MPa. Coincidentally, Grünewald *et al*. [4] tested 15 mm thick plates, recommending the improvement of their flexural strength and stiffness to satisfy admissible deformations, either by increasing thickness or introducing ribs.

Figure 8. Exploded view of the double-sided mold.

4.2. Digital fabrication of formwork

A double-sided mold made from EPS foam and timber was used to fabricate the floor-system prototype.

CNC techniques were used to mill and wirecut foam, using a home-built CNC wire-cutter and custom control software (Rippmann and Block [6]), to shape the vault and ribs and a latex based coating was applied afterwards for demolding and surface finishing. The upper foam pieces were glued on a wooden, also CNC milled frame, whereas the lower pieces were placed in a wooden case that also served as lateral formwork (Figure 8 and 9).

Figure 9: CNC milling and wire cutting to fabricate the prototype.

5. Discussion and future work

The fabrication of the prototype required research on the recipe of the concrete to find the needed flowability and compaction as the thickness of the system elements did not allow the insertion of a vibrator. Therefore, as mentioned before, the structural analysis to verify the adequacy of the topology, shape and size of the system, was carried out with assumed material properties of a lower strength concrete than the one that was finally used to build the prototype, which implies better results for the actual prototype. To compare the numerical and experimental results, future work will involve material testing of samples of the concrete that were produced during the casting of the prototype to find out the material properties to be introduced in the numerical model.

Modelling strategies will be implemented to force compressive solutions. Boundary conditions on the whole height of the corners will not allow outwards displacements, making different compressive force flows along the ribs possible.

Load tests will be carried out on the prototype, replicating the specific loading states studied on the virtual model. The tests will seek to demonstrate the success of the floor system, but also to offer the possibility to be compared with the ones from the numerical analysis, drawing conclusions about different modelling strategies, in particular in relationship to vaulted structural systems.

Related research will investigate flexible approaches to flexible bespoke pre-fabrication strategies for this system.

Figure 10. Built prototype of the ultra-thin, funicular concrete floor system.

6. Conclusion

This paper presented the process of developing a prototype for a lightweight floor system in concrete. A multi-facetted approach has been used: historical, numerical and experimental. The three perspectives are essential and complementary. History has been the inspiration and the source of knowledge that allowed the innovation. The numerical approach made the optimization processes and verification possible. The experimental approach was needed to prove the feasibility of the prototype, which is the first step towards its application in the real building.

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References

[1] Adriaenssens S., Block P., Veenendaal D., and Williams C. (eds.), *Shell Structures for Architecture. Form Finding and Optimization*. Routledge, London, 2014.

[2] BLOCK Research Group, RhinoVAULT - Designing funicular form with Rhino, 2014. http://block.arch.ethz.ch/brg/tools/rhinovault

[3] Davis L. and Block P., Earthen Masonry Vaulting: Mexican and Catalan Techniques. In: *Building SUDU: How to construct a sustainable urban dwelling unit*. Hebel D. and Moges M. (eds.), Ruby Press, Berlin, 2014.

[4] Grünewald S., Shionaga R. and Walraven J., Self-compacting Fibre Reinforced Concrete Applied in Thin Plates, in: *Rheology and processing of Construction Materials – 7th RILEM International Conference on Self-Compacting Concrete and 1st RILEM International Conference on Rheology and Processing of Construction Materials*. Roussel N., and Bessaies-Bey H. (eds.), 2013, 341-348.

[5] Ochsendorf J., *Guastavino Vaulting, The Art of Structural Tile*. Princeton Architectural Press, New York, 2010.

[6] Rippmann M. and Block P., Digital Stereotomy: Voussoir geometry for freeform masonry-like vaults informed by structural and fabrication constraints, in: *Proceedings of the IABSE-IASS Symposium 2011*, London, UK, 2011.

[7] Rippmann M., Lachauer L. and Block P., Interactive Vault Design, *International Journal of Space Structures*, 2012; *27*(4); 219-230.

[8] Rippmann M. and Block P., Constrained Funicular Modelling – Local force bounds, non-uniform loading, in: *Proceedings of the IASS-SLTE 2014 Symposium*, Brasilia, Brazil, 2014.