Geometry-based Understanding of Structures

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Summary

The development of structures with complex, curved geometry typically consists of a series of iterative steps in which formal and structural considerations are addressed separately. Since such a sequential process often results in solutions that fail to realise the aesthetic and structural intentions of the original design, there is clearly a need for an integrated approach providing bidirectional control over both form and forces at every stage of the design exploration. This paper therefore describes and advocates an approach based on elements of graphic statics, in which the design of structures is based on geometrical rather than analytical or numerical representations of the relation between form and forces. The presented approach adopts the key principles of graphic statics and extends them with the latest research on form finding, structural design and optimisation techniques.

Keywords: Structural geometry; structural equilibrium; interactive graphic statics; Thrust Network Approach; eQUILIBRIUM; GeoStat; RhinoVAULT

1. Introduction

The design and construction of architectural forms that are both expressive and structurally efficient is not a straightforward process. It requires a profound understanding of the relation between the geometry of structures and their behaviour and puts the knowledge and skills of any design and engineering team to the test.

The development of expressive structures, with complex, curved geometry, often consists of sequential steps of design and post-rationalization. Since such a twofold process often results in solutions that fail to realise the full aesthetic and structural potential of a unified spatial and structural scheme, there is a clear need for integrated approaches, providing continuous bidirectional control over both form and forces throughout the design exploration.

This paper, therefore, describes and advocates an approach in which the design of structures is based on geometrical rather than analytical or numerical representations of the relation between form and forces. This approach adopts key principles of graphic statics and extends them with insights from the latest research on form finding, structural design and optimisation. The resulting methods for two- and three-dimensional equilibrium design are implemented in modern drawing and modelling software (and in some cases integrated in web-based environments) to create tools that provide advanced understanding of the behaviour of structures and intuitive control over the complex relation between the geometry of structures and the forces in them.

2. A Geometry-based, Interactive Approach

The fundamental principles for an intuitive, integrated approach can be found in graphic statics, which is a powerful method for analysis and design of two-dimensional structures. Its strength lies in the clear representation of the relation between form and forces with simple diagrams linked through simple geometric constraints: a form diagram representing the geometry of the structure, reaction forces and applied loads; and a force diagram representing the equilibrium of the forces acting on and in the structure. With these diagrams, graphic statics provides visual information from which the behaviour of a structural system can be intuitively understood. Furthermore, since no programming skills or intimate knowledge of mathematics are required, sophisticated structural designs can be made or complex analyses can be performed by a non-specialist user with accurate results, particularly when using CAD. Most importantly, it is an integrated approach combining design and analysis into one methodology and graphic construction.

The process of making graphic statics constructions demands drafting skills and can be tedious and time-consuming though, explaining why it was pushed away by numerical methods for long. But, by implementing graphic statics-related functionality in dynamic, parametric software, such as GeoGebra [1] or Grasshopper [2], graphic statics constructions can be created that can be changed dynamically to interactively explore the relation between form and forces with real-time visual feedback, while using similar geometric construction techniques as when creating those constructions on paper (using pencil, ruler and compass). In such a setup, the bidirectional relation between form and forces, which is an inherent property of graphic statics, can be fully exploited, allowing users to intuitively steer their designs based on their appreciation of both objective and subjective criteria contained within one geometrical construct.

Furthermore, using 3D parametric CAD environments and numerical solving procedures, the two-dimensional concepts described above can be extended to three-dimensional problems, while maintaining the same clarity, rigor and intuitive visual interface, control and feedback.

In the next two sections, the relevance and power of the suggested approach will be demonstrated through specific exemplary applications for non-straightforward 2D or 3D structural problems.

3. Interactive Graphic Statics

As a clear illustration of the benefits of interactive graphic statics, consider the problem of designing a truss with a constant-force bottom chord for a given set of loads applied to a top chord with given geometry [3]. For simplicity, the applied loads are considered vertical, equal and evenly spaced along the chord.

First, the top chord is chosen to be straight. Figure 1 shows the form (left) and force (right) diagrams of this system. The lengths of the segments in the force diagram represent the magnitude of the forces in the corresponding elements in the form diagram. Using Bow's notation (labelling spaces in the form diagram and nodes in the force diagram, with the nodes in the latter corresponding to the spaces in the former), the elements of the truss can be named (e.g.: b-2 is the element separating spaces B and 2 in the form diagram and connecting nodes b and 2 in the force diagram) and grouped in the following categories:

- a-b, b-c, c-d, d-e, e-f : (magnitude of) the applied loads
- a-1, b-2, c-3, d-4, e-5, f-6 : (forces in) the members of the top chord
- g-1, g-2, g-3, g-4, g-5, g-6 : (forces in) the members of the bottom chord
- 1-2, 2-3, 3-4, 4-5, 5-6 : (forces in) the connecting struts

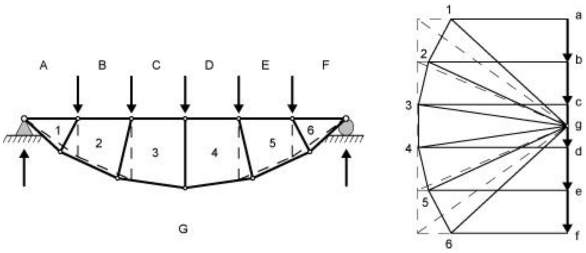

Fig. 1: Graphic statics solution to the problem of designing a truss with constant-force chords. Two solutions are shown: constant-force top chord (dashed lines), constant-force bottom chord (continuous lines). [3]

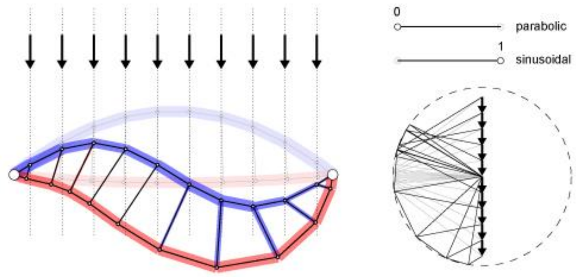

Fig. 2: Implementation of constant-force bottom chord principle in eQUILIBRIUM [4]. Simple sliders provide control over the shape of the top chord allowing for an intuitive exploration of freeform constant force trusses.

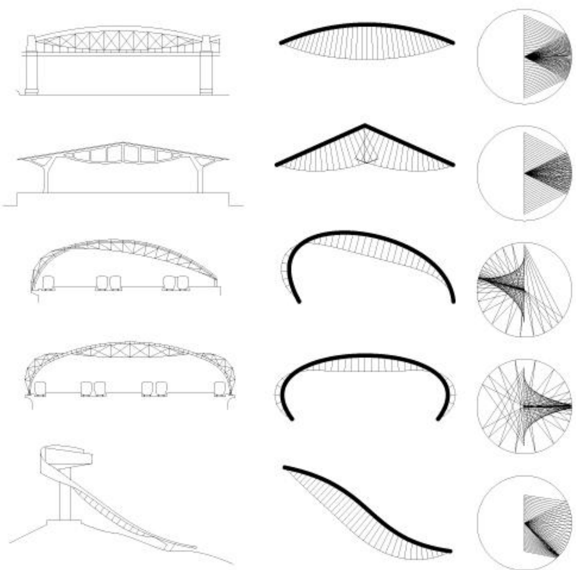

Fig. 3: Interactive, parameter-driven explorations of the relation between form and forces demonstrate how different structures can be based on the same structural principles. [5]

The first solution (dashed segments) in Figure 1 has a constant force in the top chord, i.e. the lengths of the corresponding segments in the force diagram are equal, resulting in vertical connecting struts. The second solution (continuous segments) has a constant force in its bottom chord. There, the requirement that all elements in the force diagram corresponding to the bottom chord need to have the same length is fulfilled by constraining points 1 to 6 to a circle with its centre at *g* and as radius the requested magnitude of force in the chord. This geometric constraint shows that the struts in this solution are not vertical, and allows their required orientation to be derived directly from the force diagram.

Solving this problem using numerical or analytical methods is not straightforward. With graphic statics, however, it is as simple as constraining elements in the force diagram to a circle.

Furthermore, as seen in Figure 2, once such a construction has been set up in an interactive drawing environment as described in Section 2, an entire range of freeform constant-force trusses can be designed and evaluated in an intuitive and exploratory manner using simple controls, such as b-spline control points or shape superposition sliders (Fig. 2) [4].

Moreover, such explorations can lead to new insights, such as the realisation that the formally very different structures shown in Figure 3 could be designed using the same structural concept, i.e. with the same geometric constraints applied to their force diagrams [5].

4. Bidirectional Thrust Network Approach

This section shows how the previous described concept can be used for complex three-dimensional systems. The Thrust Network Approach (TNA) [6,7] was initially developed to generate funicular compression-only networks. Because of its graphic statics-based solving strategy, it provides an intuitive and visual way to generate equilibrium shapes. TNA can be seen as a three-dimensional extension of graphic statics (for the case of parallel loading). Although the highly indeterminate 3D networks and the force equilibrium in space are significantly more complex than in two-dimensional problems, the inherent simplicity and intuitive readability of the graphical representation

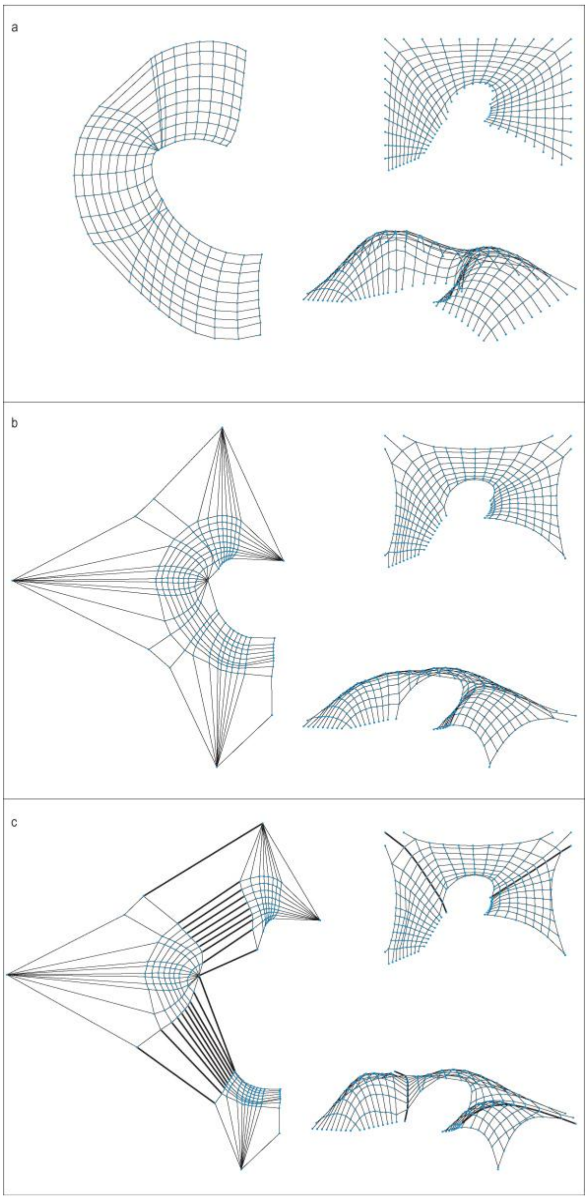

Fig. 4: Overview of TNA-based design process of a funicular compression-only vault with RhinoVAULT [10]: a)starting geometry, b) integration of open edges, c) final solution after re-distribution of internal forces.

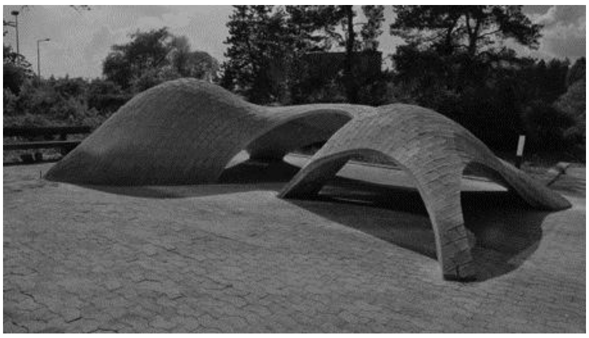

Fig. 5: Prototype of freeform unreinforced masonry vault at ETH Zurich. [11]

of form and forces through geometrically linked diagrams results in a deeper understanding of the structural form, especially in comparison to analytical and numerical approaches. Moreover, this approach allows the flexible generation of freeform-like compression-only networks with complex geometry [8].

Because the loading is constrained to vertical loading in TNA (which is a reasonable assumption in the design stages), two dimensional diagrams can be used to shape the 3D equilibrium network: the horizontal projection of the network serves as form diagram, linked with a geometrical, reciprocal relation to a force diagram [9,6]. Hence, the in-plane equilibrium of the form diagram, which is the horizontal equilibrium of the equilibrium network, is represented, visualized, and controlled by the force diagram.

Very much like the circle constraint (i.e. equal lengths constraint) on the force diagram of the constant force truss in Section 3, the compression-only requirement on the 3D network can now be enforced geometrically by requiring the form and force diagram not only to be reciprocal, but also convex for each of the individual cell polygons [8]. Implementing these constraints ("convex reciprocal") in an interactive and bidirectional manner then allows to understand the relation between force pattern (= form diagram) and force distribution (= force diagram) respectively the 3D equilibrium shape. As an example, this correlation is shown in Figure 4c where higher forces along certain lines in the form diagram (thicker lines), resulting from, but also represented by the stretched elements in the force diagram, generate a kink in the 3D equilibrium network, or structurally speaking, attract more forces along those "ribs".

This approach has a clear relevance for teaching (providing high-level insights), practice (producing unique funicular structures, Fig. 5 [11]), and research (developing new insights in 3D equilibrium in general [12,13])

5. Implementations

Interactive applications have been developed by the authors which were used to generate the examples in Sections 3 and 4. GeoStat, a custom-made, graphic statics-specific parametric drawing environment, based on GeoGebra, was used to produce the examples in Section 3. The interactive drawings can be found on eQUILIBRIUM, which is an interactive, web-based learning platform for structural design, used for the introductory structures classes in architecture at ETH Zurich [4]. The examples in Section 4 were produced using RhinoVAULT, which is a plugin for Rhinoceros for TNA form finding of vaults and shells [10]. Both tools are currently in alpha development.

6. Conclusions

The presented approach, and the two- and three-dimensional implementations, bring high level structural engineering concepts to students, resulting in a very steep learning curve. The tools furthermore allow for powerful exploration of structural form, while providing new, and deeper insights into complex equilibrium. An important aspects is that the same tools pushing research are also used for novel practical implementations, as well as projects and teaching at an introductory level. This paper argued that successful tools have relevance in teaching, research and practice.

As a final conclusion, this paper thus also made it clear that the core topics of the Structural Morphology Group (SMG), to deepen the understanding of the relations between structures and geometry, have a large relevance in the next generation of research, practical and teaching tools.

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